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In[186]:= radcan[z_] := ToRadicals[RootReduce[FunctionExpand[{Re[z], Im[z]}]]].{1, I}

Clear[pentfill]; pentfill[t_, a_: 1, b_: 0] :=
pentfill[t, foo_: 69, bar_: 105] = radcan[pentfill[t, s1_: 1, s0_: 0] = ((b - s0) / (s1 - a));

Block[{A = radcan[a], B = radcan[b], u = Expand[ $\frac{1}{2} (5 + 3 \sqrt{5}) t$ ], T},
T = radcan[Mod[u, 1]]; pntfiln[Floor[u] + Floor[Expand[u +  $\frac{1}{2} (5 - 3 \sqrt{5})$ ]]]]]]

In[2]:= pntfiln[-1] := Block[{a = E^(3 * 2 * I *  $\pi$  / 5) / GoldenRatio^2, b = E^(3 * I *  $\pi$  / 10) / GoldenRatio},
a * pentfill[ $1 - \frac{1}{10} (5 + 3 \sqrt{5}) * T$ , a * A, B - A * b] - b]

In[3]:= pntfiln[0] := Block[{a = E^(8 * I *  $\pi$  / 5) / GoldenRatio^2},
a * (trifill[ $1 - \left(T - \frac{1}{2} (-5 + 3 \sqrt{5})\right) * \frac{1}{2} (7 + 3 \sqrt{5})$ , a * A, B - a * A * I] - I)]

In[99]:= pntfiln[1] := Block[{a = E^(3 * 2 * I *  $\pi$  / 5) / GoldenRatio^2, b = E^(9 * I *  $\pi$  / 10) / GoldenRatio},
a * pentfill[ $1 - \frac{1}{10} (5 + 3 \sqrt{5}) * T$ , a * A, B + A * b] + b]

In[76]:= pntfiln[2] := Block[{a = E^(6 * I *  $\pi$  / 5) / GoldenRatio^2},
a * (irtfill[ $\left(T - \frac{1}{2} (-5 + 3 \sqrt{5})\right) * \frac{1}{2} (7 + 3 \sqrt{5})$ , a * A, B - a * A * I] - I)]

In[6]:= pntfiln[3] := Block[{a = 1 / GoldenRatio^2, b = I / GoldenRatio},
a * pentfill[ $\frac{1}{10} (5 + 3 \sqrt{5}) * T$ , a * A, B + A * b] + b]

In[77]:= pntfiln[4] := Block[{a = E^(4 * I *  $\pi$  / 5) / GoldenRatio^2},
a * (irtfill[ $\left(T - \frac{1}{2} (-5 + 3 \sqrt{5})\right) * \frac{1}{2} (7 + 3 \sqrt{5})$ , a * A, B - a * A * I] - I)]

In[8]:= pntfiln[5] :=
Block[{a = E^(I *  $\pi$  / 5) / GoldenRatio^2}, a * pentfill[ $1 - \frac{1}{10} (5 + 3 \sqrt{5}) * T$ , a * A, B]]

In[9]:= pntfiln[6] := Block[{a = 1 / GoldenRatio^2},
a * (irtfill[ $1 - \left(T - \frac{1}{2} (-5 + 3 \sqrt{5})\right) * \frac{1}{2} (7 + 3 \sqrt{5})$ , a * A, B - a * A * I] - I)]

In[10]:= pntfiln[7] := Block[{a = 1 / GoldenRatio^2, b = E^(7 * I *  $\pi$  / 10) / GoldenRatio},
a * pentfill[ $\frac{1}{10} (5 + 3 \sqrt{5}) * T$ , a * A, B - A * b] - b]

In[11]:= pntfiln[8] := Block[{a = E^(2 * I *  $\pi$  / 5) / GoldenRatio^2},
a * (trifill[ $\left(T - \frac{1}{2} (-5 + 3 \sqrt{5})\right) * \frac{1}{2} (7 + 3 \sqrt{5})$ , a * A, B - a * A * I] - I)]

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In[12]:= pntfiln[9] := Block[{a = 1 / GoldenRatio^2, b = E^(I * π / 10) / GoldenRatio},
  a * pentfill[ $\frac{1}{10} (5 + 3 \sqrt{5}) * T, a * A, B + A * b] + b]$ 
```

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In[13]:= pntfiln[10] = E^(I * π / 10)
Clear[trifill]; trifill[t_, a_: 1, b_: 0] := trifill[t, x_: 0, y_: 0] =
  radcan[trifill[t, s1_: 1, s0_: 0] = ((b - s0) / (s1 - a)); Block[{A = a, B = b, T = Simplify[t]},
  Which[T <  $\frac{1}{8} (3 - \sqrt{5})$ , Block[{c =  $\frac{1}{2} \left( -\frac{3}{4} + \frac{\sqrt{5}}{4} - \frac{1}{2} i \sqrt{\frac{5}{2} - \frac{\sqrt{5}}{2}} \right)$ , d =  $\frac{i \sqrt{5}}{2}$ },
    c * irtfill[2 * T * (3 +  $\sqrt{5}$ ), c * A, B - d * a] - d],
  T <  $\frac{1}{4} (3 - \sqrt{5})$ , Block[{c =  $\frac{1}{2} \left( -\frac{3}{4} + \frac{\sqrt{5}}{4} + \frac{1}{2} i \sqrt{\frac{5}{2} - \frac{\sqrt{5}}{2}} \right)$ , d =  $\frac{i \sqrt{5}}{2}$ },
    c * irtfill[1 - 2 * (T -  $\frac{1}{8} (3 - \sqrt{5})$ ) * (3 +  $\sqrt{5}$ ), c * A, B - d * a] - d],
  T < 3 / 4, Block[{c =  $\frac{1}{4} (-1 + \sqrt{5}) * E^{(3 * I * \pi / 5)}$ , d =  $\frac{1}{2} i \sqrt{\frac{1}{2} (3 + \sqrt{5})}$ },
    c * pentfill[(T -  $\frac{1}{4} (3 - \sqrt{5})$ ) / Sqrt[5] * 4, c * A, B - d * A] - d],
  True, trifill[4 * T - 3, A / 2, B] / 2]]]
Clear[irtfill]; irtfill[t_, a_: 1, b_: 0] := irtfill[t, x_: 0, y_: 0] =
  radcan[irtfill[t, s1_: 1, s0_: 0] = ((b - s0) / (s1 - a)); Block[{A = a, B = b, T = Simplify[t]},
  Which[T <  $\frac{1}{8} (3 - \sqrt{5})$ , Block[{c =  $\frac{1}{2} \left( -\frac{3}{4} + \frac{\sqrt{5}}{4} + \frac{1}{2} i \sqrt{\frac{5}{2} - \frac{\sqrt{5}}{2}} \right)$ , d =  $\frac{i \sqrt{5}}{2}$ },
    c * trifill[2 * T * (3 +  $\sqrt{5}$ ), c * A, B - d * a] - d],
  T <  $\frac{1}{4} (3 - \sqrt{5})$ , Block[{c =  $\frac{1}{2} \left( -\frac{3}{4} + \frac{\sqrt{5}}{4} - \frac{1}{2} i \sqrt{\frac{5}{2} - \frac{\sqrt{5}}{2}} \right)$ , d =  $\frac{i \sqrt{5}}{2}$ },
    c * trifill[1 - 2 * (T -  $\frac{1}{8} (3 - \sqrt{5})$ ) * (3 +  $\sqrt{5}$ ), c * A, B - d * a] - d],
  T < 3 / 4, Block[{c =  $\frac{1}{4} (1 - \sqrt{5})$ , d =  $\frac{1}{2} i \sqrt{\frac{1}{2} (3 + \sqrt{5})}$ },
    c * pentfill[1 - (T -  $\frac{1}{4} (3 - \sqrt{5})$ ) / Sqrt[5] * 4, c * A, B - d * A] - d],
  True, irtfill[4 * T - 3, A / 2, B] / 2]]]

```

In[247]:= **pentfill**[0], **pentfill**[1]

$$\text{Out[247]} = \left\{ \frac{1}{4} i \left(-1 - \sqrt{5} \right) - \frac{1}{2} \sqrt{\frac{1}{2} \left(5 - \sqrt{5} \right)}, \sqrt{\frac{5}{8} + \frac{\sqrt{5}}{8}} + \frac{1}{4} i \left(-1 + \sqrt{5} \right) \right\}$$

In[248]:= **absarg**[z_] := **FullSimplify**[{**Abs**[z], **Arg**[z]}]; **absarg** /@ %

$$\text{Out[248]} = \left\{ \left\{ 1, -\frac{7\pi}{10} \right\}, \left\{ 1, \frac{\pi}{10} \right\} \right\}$$

In[187]:= **pentfill**[1 / 2]

$$\text{Out[187]} = \frac{7}{44} \sqrt{\frac{1}{2} \left(85 - 31 \sqrt{5} \right)} + \frac{1}{88} i \left(-57 + 49 \sqrt{5} \right)$$

In[139]:= **pentfill**[73 / 100]

$$\begin{aligned} \text{Out[139]} = & \frac{1}{468\,604\,930\,048} \left(\sqrt{\left(39\,949\,909\,581\,150\,570\,569\,846\,991\,782\,179\,253\,334\,521 - \right. \right. \\ & 17\,866\,142\,703\,684\,565\,370\,634\,037\,454\,670\,015\,407\,340 \sqrt{5} - 6 \\ & \left. \sqrt{\left(-10\,697\,030\,450\,563\,301\,641\,869\,967\,734\,810\,257\,202\,527\,549\,650\,588\,998\,572\,827\,588\,941\,520\,256 - \right. \right. \\ & 850\,785 + \\ & 4\,783\,857\,448\,968\,949\,202\,828\,735\,200\,033\,213\,692\,702\,983\,550\,707\,316\,585\,182\,127\,811\,538\,699\,891 \\ & \left. \left. \left. 962 \sqrt{5} \right) \right) \right) \end{aligned}$$

In[140]:= **N**[%]

$$\text{Out[140]} = 0. + 5885.3 i$$

In[141]:= **N**[%%, 105]

$$\begin{aligned} \text{Out[141]} = & 0.2131374408193970964741771834512439166744742270659357898526845475749997415236859224210344 \\ & 90087569772038068 - \\ & 0.238051418014799443843113948668971401037609401316847697211297434591611104003563228432636 \\ & 571978459919764008 i \end{aligned}$$

In[142]:= **N**[%%%, 9]

$$\text{Out[142]} = 0.213137441 - 0.238051418 i$$

In[252]:= **pentfill** $\left[\frac{1}{10} \left(-5 + 3 \sqrt{5} \right)\right]$

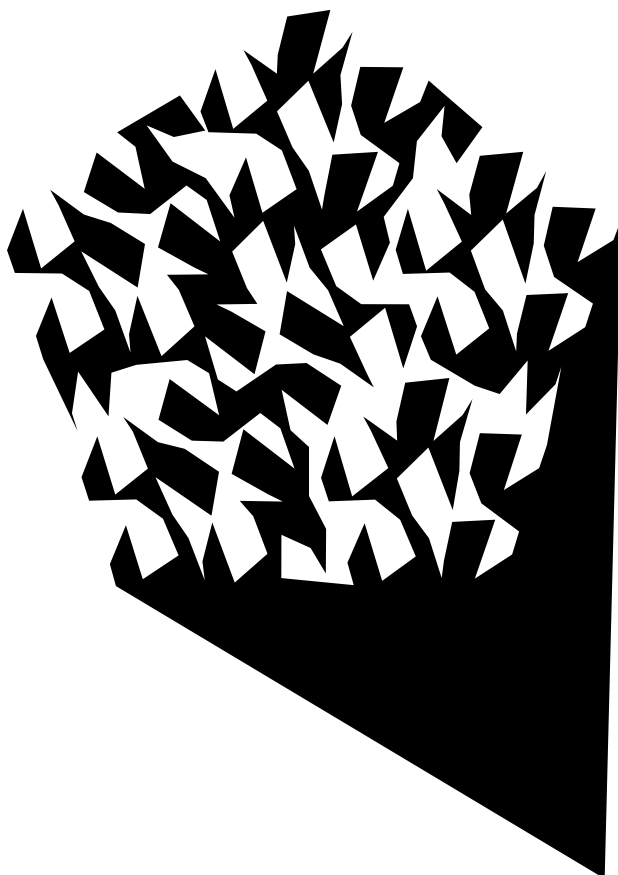
$$\text{Out[252]} = -\frac{1}{2} \sqrt{5 \left(5 - 2 \sqrt{5} \right)} + \frac{1}{2} i \left(2 - \sqrt{5} \right)$$

In[250]:= **pentfill**[1 / **GoldenRatio**]

$$\text{Out[250]} = -\frac{1}{22} \sqrt{\frac{5}{2} \left(205 - 89 \sqrt{5} \right)} + \frac{1}{44} i \left(19 - 9 \sqrt{5} \right)$$

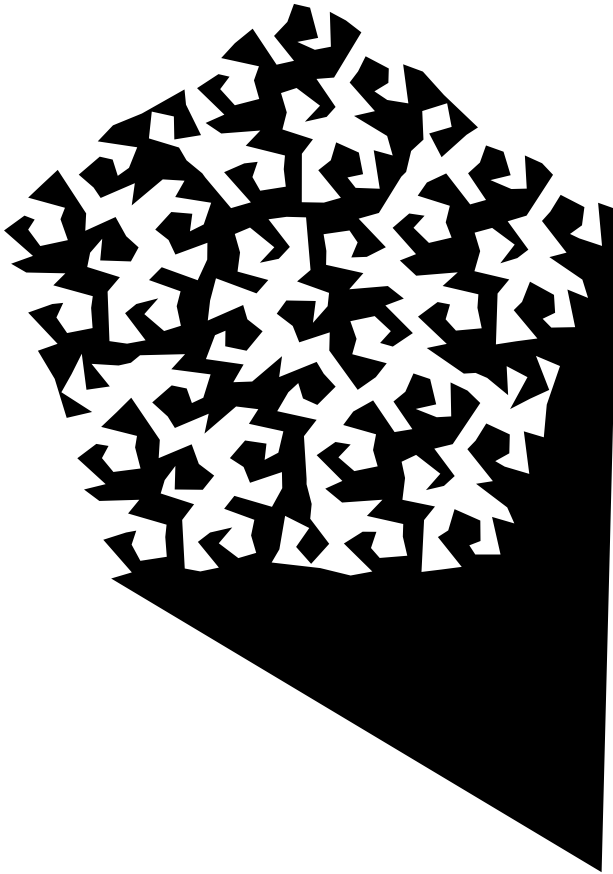
```
In[249]:= (Print[#1]; #2) &@@ AbsoluteTiming[Graphics[  
  Polygon[{Re[#], Im[#]} & /@ Join[ {.9 - 1.7 * I}, Table[N[pentfill[k / 281], 9], {k, 0, 281}]]]]]  
1152.591642
```

Out[249]=



```
In[218]:= (Print[#1]; #2) &@ AbsoluteTiming[Graphics[
  Polygon[{Re[#], Im[#]} & /@ Join[{.9 - 1.7 * I}, Table[N[pentfill[k / 521], 9], {k, 0, 521}]]]]]
1199.304940
```

Out[218]=



```
In[211]:= (Print[#1]; #2) &@ AbsoluteTiming[
  Row[{Graphics[Polygon[{Re[#], Im[#]} & /@ Append[#, #[[1]] - Im[#[[1]]] * I / 2] &@
    Table[N[trifill[k / 21], 9], {k, 0, 21}]]],
    Graphics[Polygon[{Re[#], Im[#]} & /@ Append[#, #[[1]] - Im[#[[1]]] * I / 2] &@
    Table[N[irtfill[k / 21], 9], {k, 0, 21}]]]]]
20.052480
```

Out[211]=



```
In[246]:= (Print[#1]; #2) &@@  
AbsoluteTiming[Graphics[Polygon[{Re[#], Im[#]} & /@ Join[{.9 - 1.7 * I}, Table[  
N[pentfill[k * GoldenRatio / 233], 9], {k, 0, Floor[233 / GoldenRatio]}]]]]]  
3429.040729
```

Out[246]=

