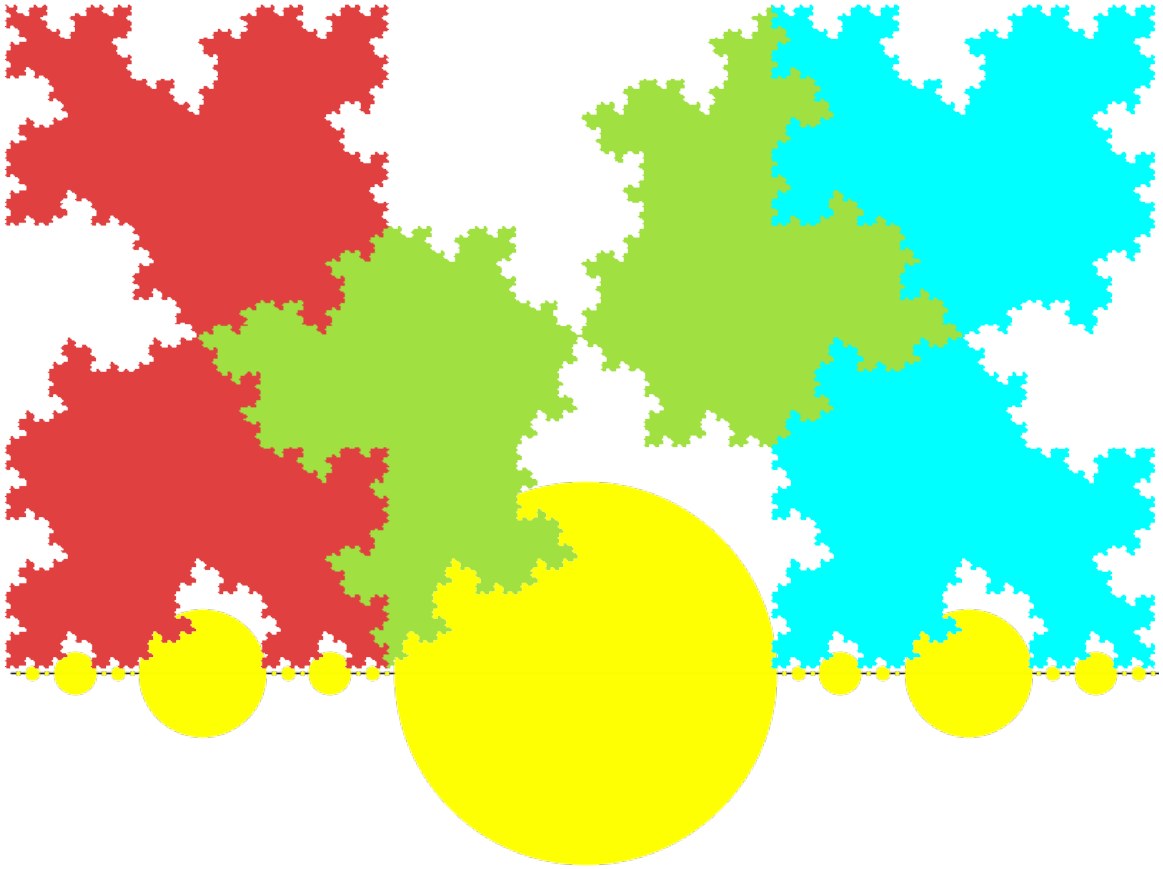
Cantor ś set



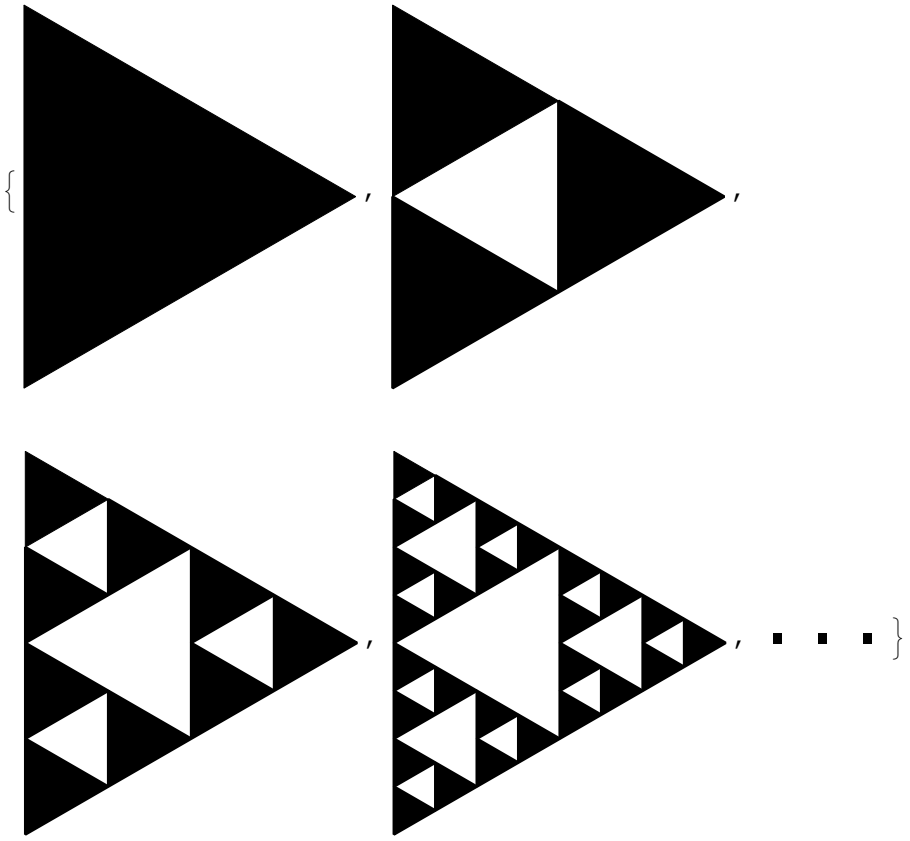
>> ⋃(disk diameter endpoints)!

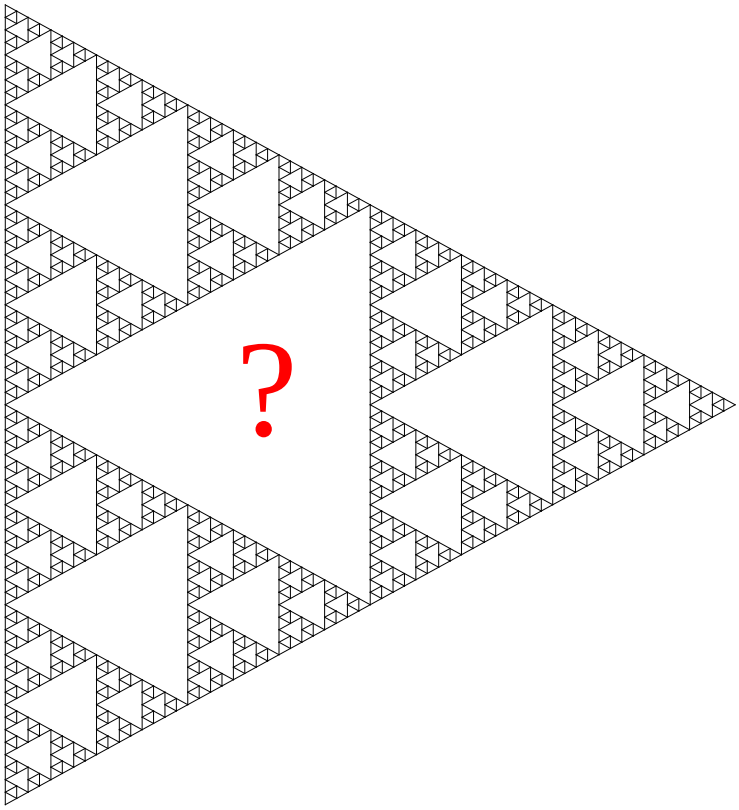
0.01101001100101 1...

→ 0.02202002200202 2...

Sierpiński ś Gasket

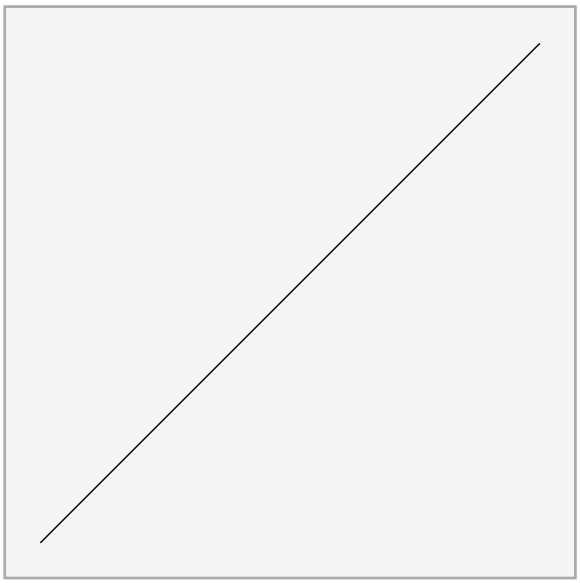
Joint work with Julian Ziegler Hunts (paper)
Neil Bickford (visuals)



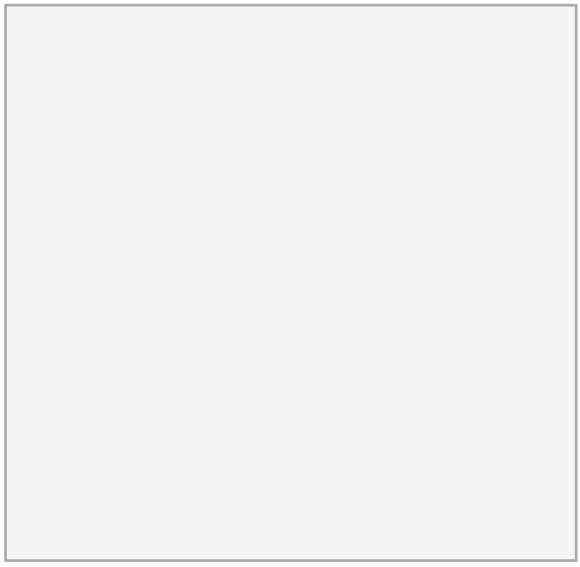


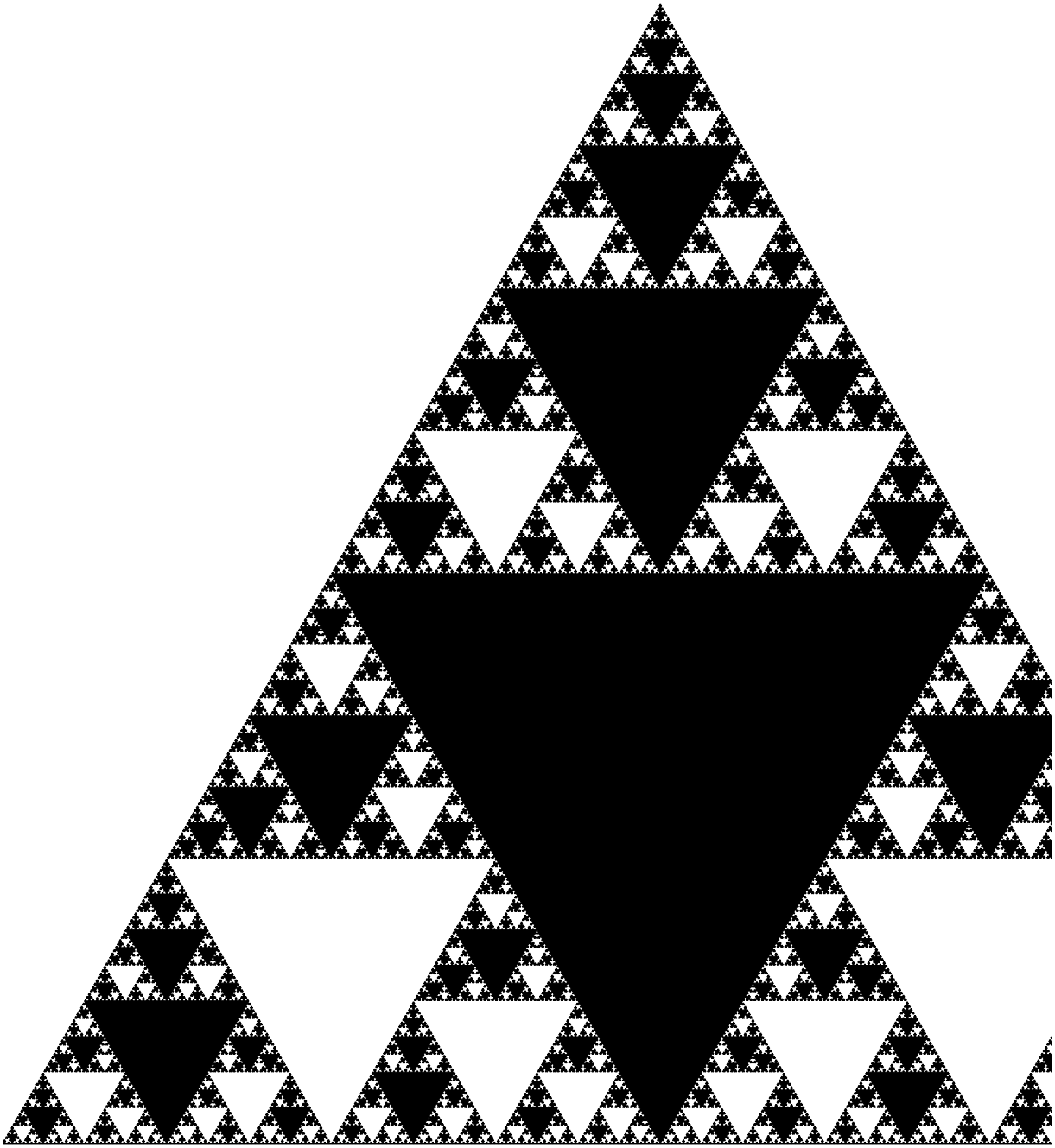
No, !

A Rectangle

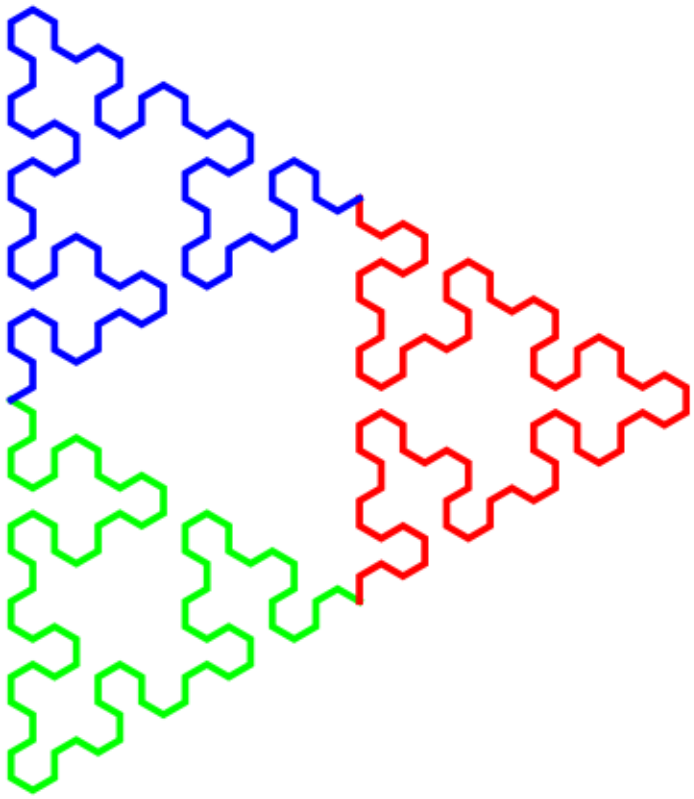


A Line Segment





```
Import["Math/Misc./Gasket paper/gasketrecursion3.png"]
```



```
Sierp[t_, a1_: 1, a0_: 0] := Sierp[t, b1_: 1, b0_: 0] =
  ComplexExpand[(Sierp[t, s1_: 1, s0_: 0] = ((a0 - s0) / (s1 - a1));
    Module[{t3 = 3 * (t + 1), n}, n = Floor[t3 / 2]; t3 -= 2 n;
     Switch[n,
      0, Exp[2 * I * π / 3] * (Sierp[1 - t3, Exp[2 * I * π / 3] * a1 / 2,
           a0 - a1 * Exp[2 * I * π / 3] * Sqrt[3] / 2] - Sqrt[3]) / 2,
      1, (Sierp[t3 - 1, a1 / 2, a0 + a1 * Sqrt[3] / 2] + Sqrt[3]) / 2,
      2, Exp[I * π / 3] * (Sqrt[3] - Sierp[1 - t3,
           -a1 * Exp[I * π / 3] / 2, a0 + a1 * Exp[I * π / 3] * Sqrt[3] / 2]) / 2, 3, I]])]
```

```
Table[Sierp[t], {t, -1, 1, 1 / 13}]
```

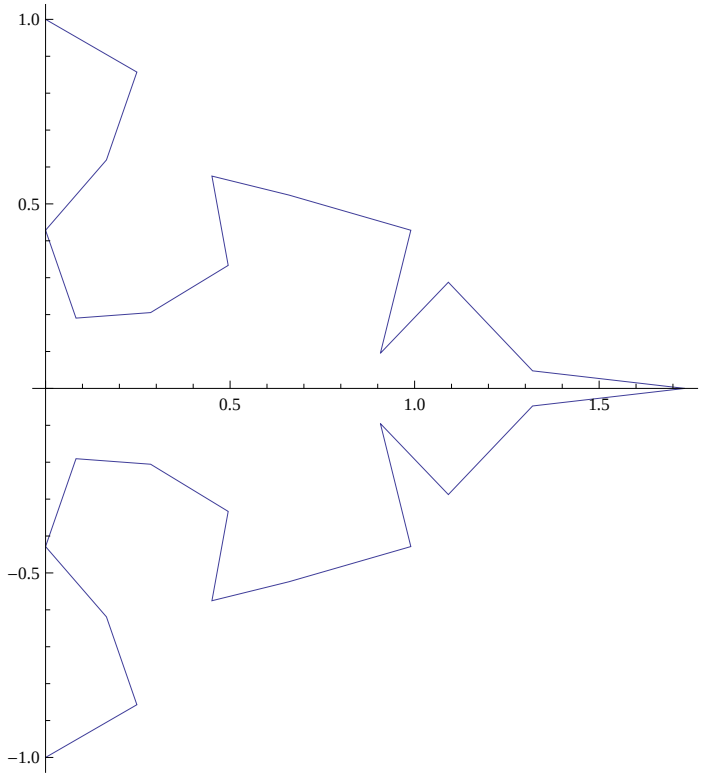
$$\left\{-i,\ -\frac{6\,i}{7}+\frac{\sqrt{3}}{7},\ -\frac{13\,i}{21}+\frac{2}{7\sqrt{3}},\ -\frac{3\,i}{7},\ -\frac{4\,i}{21}+\frac{1}{7\sqrt{3}},\ -\frac{15\,i}{73}+\frac{12\sqrt{3}}{73},\ -\frac{i}{3}+\frac{2\sqrt{3}}{7},\right.$$

$$-\frac{42\,i}{73}+\frac{19\sqrt{3}}{73},\ -\frac{11\,i}{21}+\frac{8}{7\sqrt{3}},\ -\frac{3\,i}{7}+\frac{4\sqrt{3}}{7},\ -\frac{2\,i}{21}+\frac{11}{7\sqrt{3}},\ -\frac{21\,i}{73}+\frac{46\sqrt{3}}{73},$$

$$-\frac{i}{21}+\frac{16}{7\sqrt{3}},\ \sqrt{3},\ \frac{i}{21}+\frac{16}{7\sqrt{3}},\ \frac{21\,i}{73}+\frac{46\sqrt{3}}{73},\ \frac{2\,i}{21}+\frac{11}{7\sqrt{3}},\ \frac{3\,i}{7}+\frac{4\sqrt{3}}{7},\ \frac{11\,i}{21}+\frac{8}{7\sqrt{3}},$$

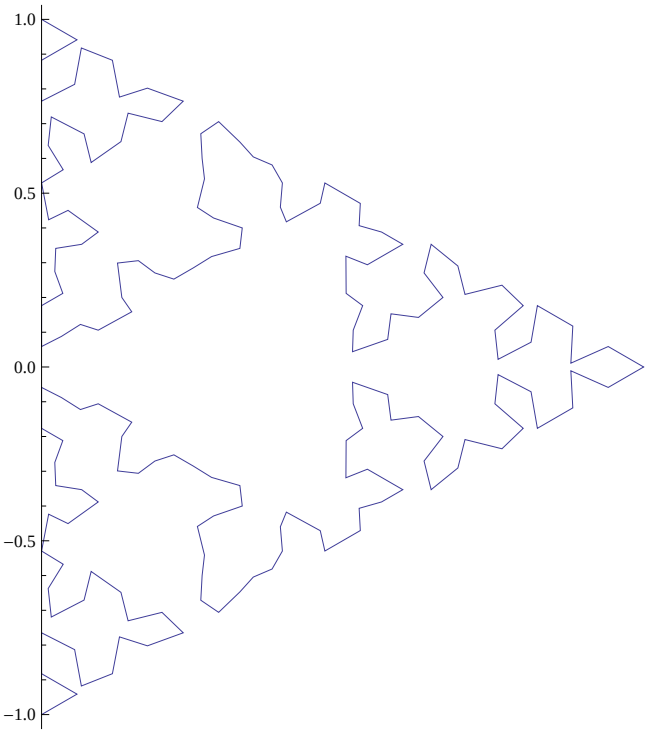
$$\left.\frac{42\,i}{73}+\frac{19\sqrt{3}}{73},\ \frac{i}{3}+\frac{2\sqrt{3}}{7},\ \frac{15\,i}{73}+\frac{12\sqrt{3}}{73},\ \frac{4\,i}{21}+\frac{1}{7\sqrt{3}},\ \frac{3\,i}{7},\ \frac{13\,i}{21}+\frac{2}{7\sqrt{3}},\ \frac{6\,i}{7}+\frac{\sqrt{3}}{7},\ i\right\}$$

```
ListLinePlot[{Re[#], Im[#]} & /@ %%, AspectRatio → Automatic]
```



```
ListLinePlot[{Re[#], Im[#]} & /@ Table[Sierp[t], {t, -1, 1, 1 / 82}],
 AspectRatio → Automatic, Axes → {False, True},
 PlotLabel → Style[{-1, -81 / 82, "...", 81 / 82, 1}, Large]]
```

$$\left\{-1, -\frac{81}{82}, \ldots, \frac{81}{82}, 1\right\}$$




```
Run["open ~/Downloads/Sierp-1.wmv"];
```

```
Sum[A[k + 1 / m] * E^(I * π * (k + 1 / m) * t), {k, -∞, ∞}]
```

$$\sum_{k=-\infty}^{\infty} A\left(k + \frac{1}{m}\right) e^{i \pi t \left(k + \frac{1}{m}\right)}$$

```
MatrixForm[{{A[x], -A[x]}, {-A[-x], A[-x]}}] ==
 HoldForm[2 / π / x] * Product[MatrixForm[
    {{1 / 2, -Sin[π / 6 + 2 * π * x / 3^n]}, {-Sin[π / 6 - 2 * π * x / 3^n], 1 / 2}}], {n, 1, ∞}]
```

$$\begin{pmatrix} A(x) & -A(x) \\ -A(-x) & A(-x) \end{pmatrix} =$$

$$\frac{2}{\pi x}\prod_{n=1}^{\infty}\begin{pmatrix} \frac{1}{2} & -\sin\left(2\times 3^{-n}\pi x+\frac{\pi}{6}\right) \\ -\sin\left(\frac{\pi}{6}-2\times 3^{-n}\pi x\right) & \frac{1}{2} \end{pmatrix}$$

```
A[x] == HoldForm[1 / π / x] *
   Limit[HoldForm[2^-n] * Sum[(-1)^k * Cos[2 * π * (d[k] / 3 + k * x / 3^n)],
      {k, -Floor[3^n / 2], Floor[3^n / HoldForm[2]]}], n → ∞]
```

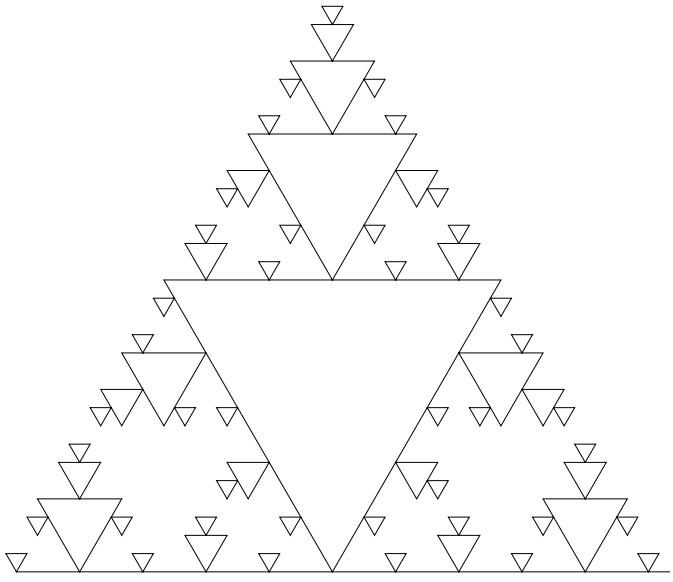
$$A(x)=\frac{1}{\pi x}\left(\lim_{n\to\infty} 2^{-n}\sum_{k=-\left\lfloor\frac{3^n}{2}\right\rfloor}^{\left\lfloor\frac{3^n}{2}\right\rfloor}(-1)^k\cos\left(2\pi\left(\frac{d(k)}{3}+k\,3^{-n}x\right)\right)\right)$$

```
Clear[d]; d[239] == 1
```

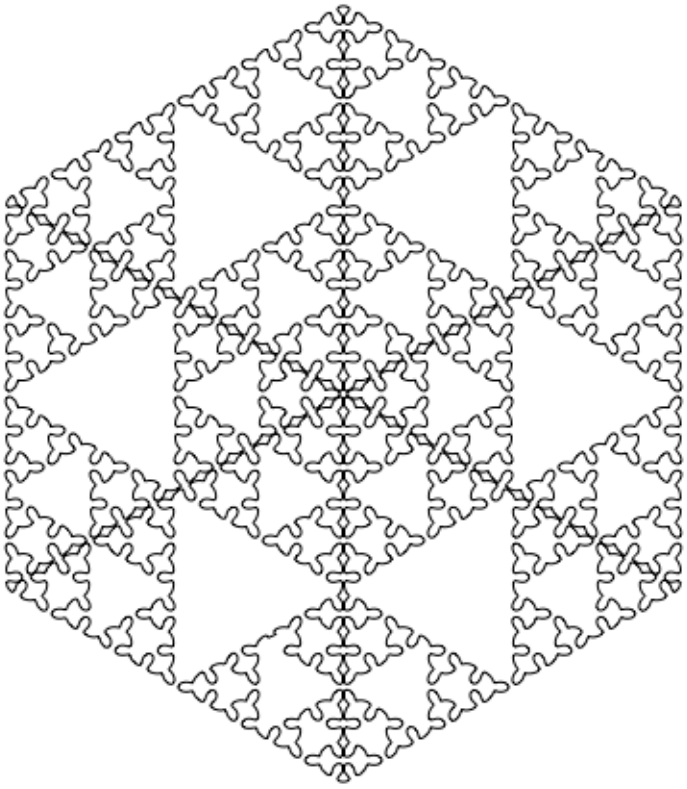
$$d(239)=1$$

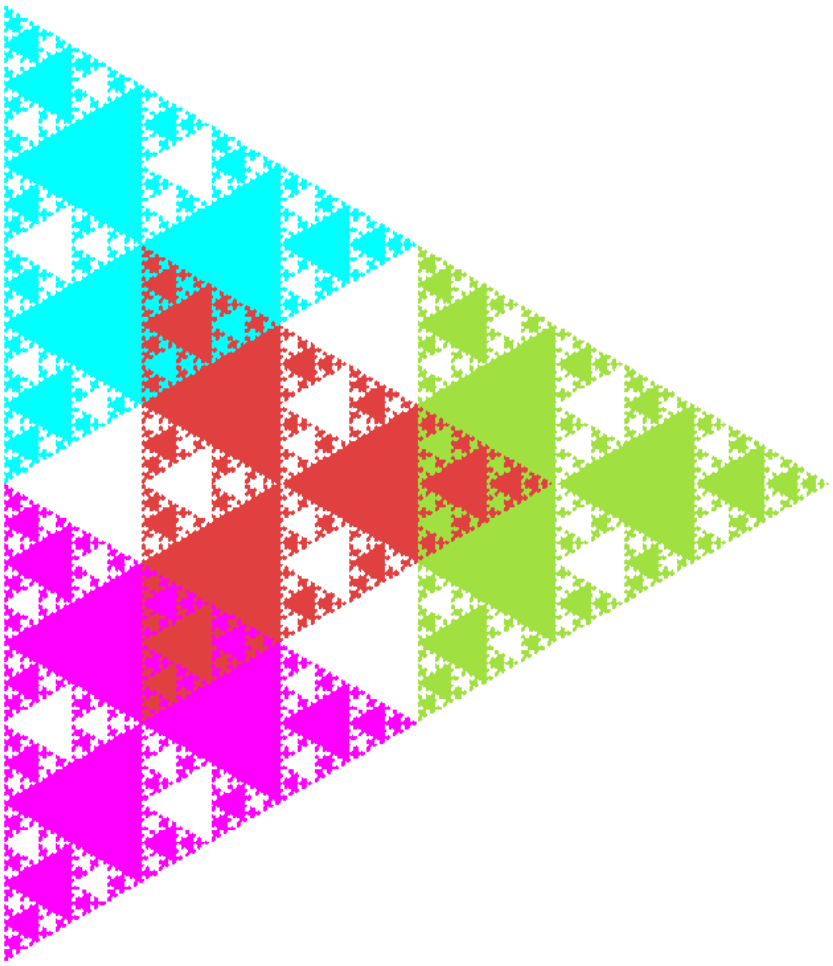
```
239_10 == 1000❶❶_3 → 1❶❶ → ❶❶1_-1 == -(-1)^2 - (-1)^1 + 1 (-1)^0 == 1
```

```
Graphics[
 Line[({Re[#1], Im[#1]} &) /@ Accumulate[Table[E^((2 / 3) * I * Pi * magicd[t]),
     {t, -2 * Floor[3^6 / 4], 2 * Floor[3^6 / 4], 2}]]]]
```



```
Import["Math/Misc./Gasket paper/fourierdoily239.png"]
```





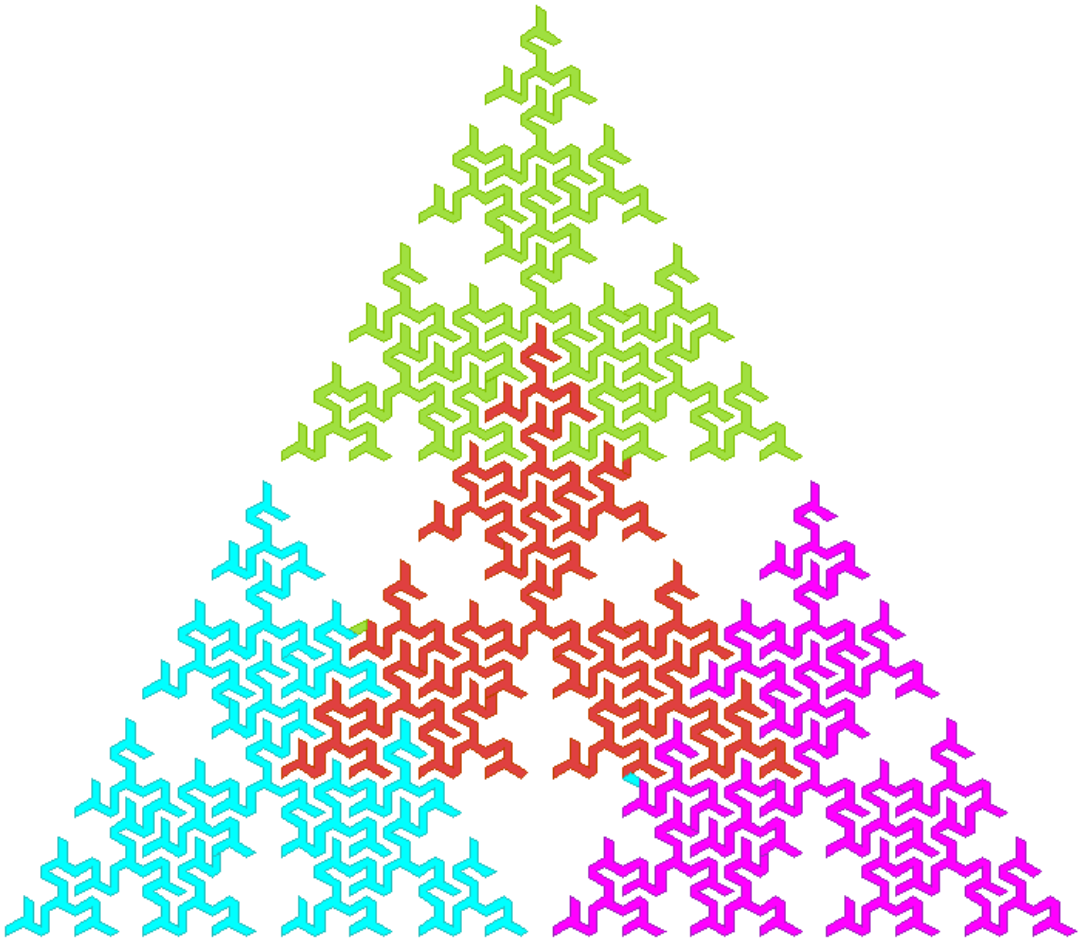
$$\sum_{k=-\infty}^{\infty} B(k,\, m)\, e^{i\,\pi\,\frac{t}{2}\left(k+\frac{1}{m}\right)}$$

$$B(k,\, m) = \frac{1}{2}\left(A\left(\frac{1}{2}\left(k+\frac{1}{m}\right)\right) - (-1)^k \overline{A\left(-\frac{1}{2}\left(k+\frac{1}{m}\right)\right)}\right)$$

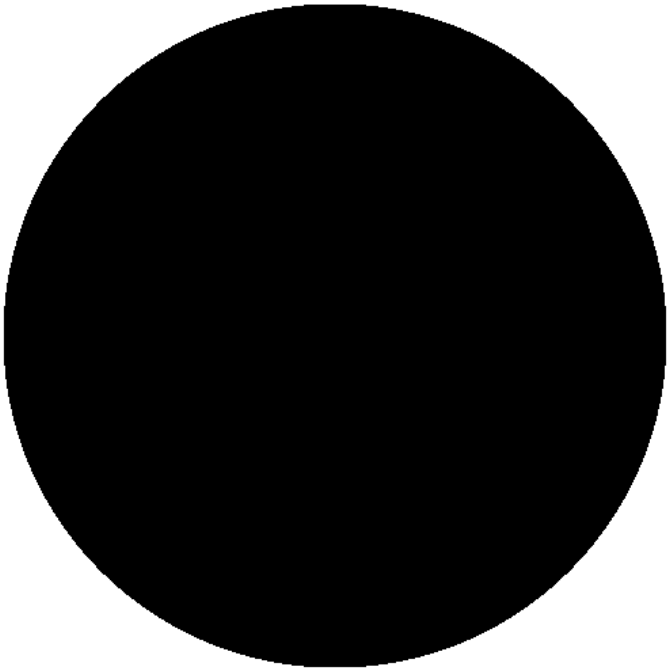
$$\pi\left(k+\frac{1}{m}\right) B(k,\, m) = \prod_{n=1}^{\infty}\left(\begin{array}{ll} (-1)^k \sin\left(\pi\left(\frac{1}{6} - 3^{-n}\left(k+\frac{1}{m}\right)\right)\right) + \frac{1}{2} & - \\ -\sin\left(\pi\left(\frac{1}{6} - 3^{-n}\left(k+\frac{1}{m}\right)\right)\right) & \frac{1}{2} - (- \end{array}\right.$$

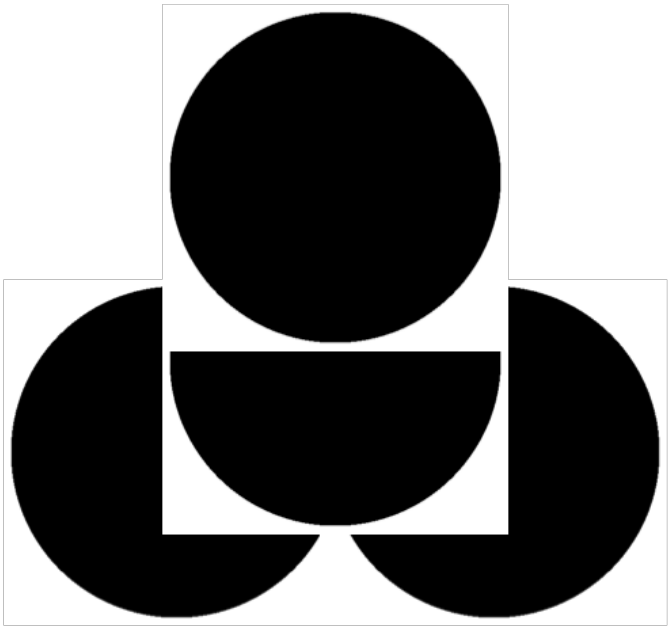
```
Run["open ~/Downloads/Base2alores-1.wmv"];
Run["open ~/Downloads/Base2hires-1.wmv"];
```

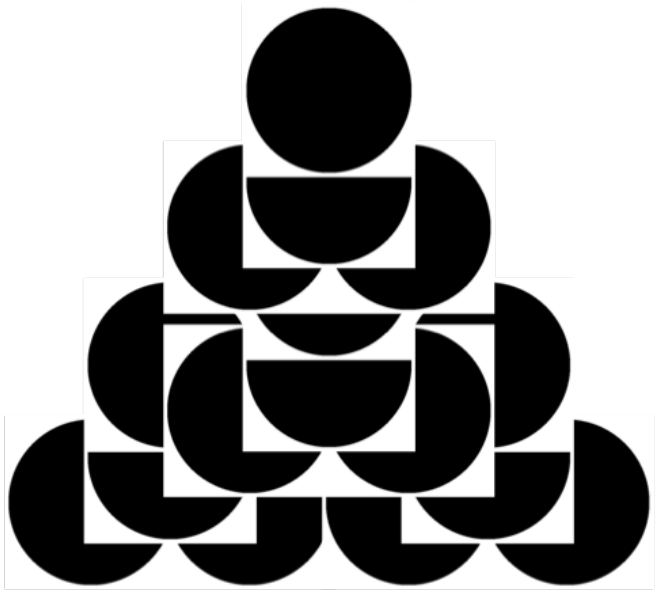
```
Import["Math/Misc./Gasket paper/bacefill.png"]
```



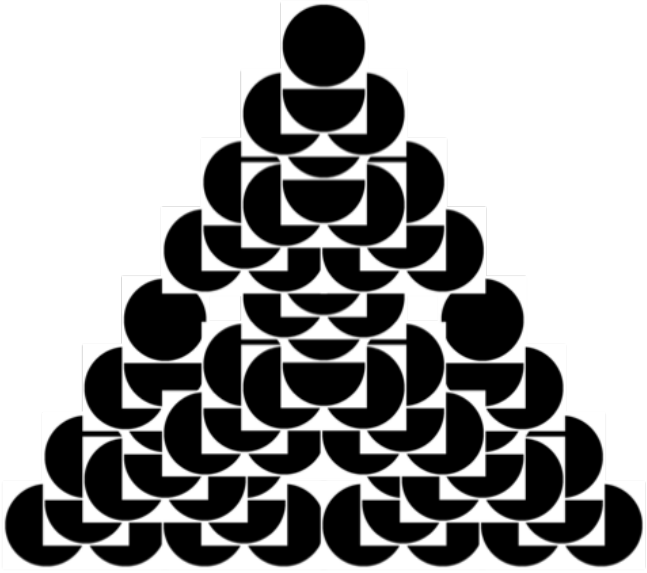
```
NestList[Rasterize[
   Style[Graphics[Table[{Translate[#[[1]], pts[[n]] * 330 / 256 * 256]}, {n, 1, 4}]],
    Antialiasing → False], RasterSize → 512, Background → None] &, %, 10]
```

{  ,

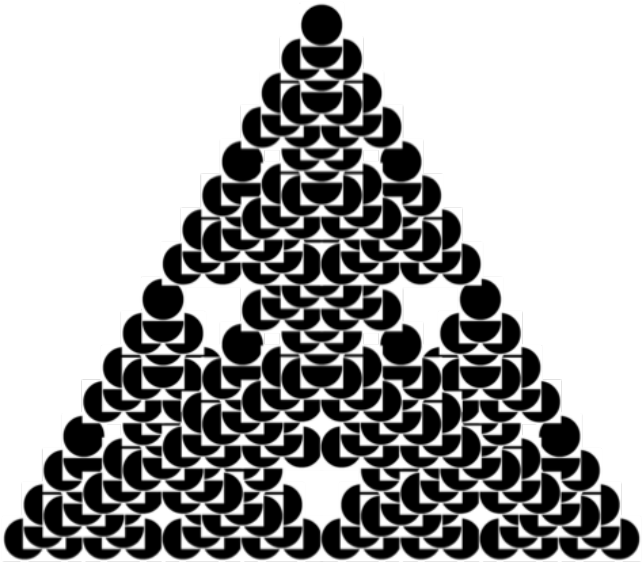
 ,



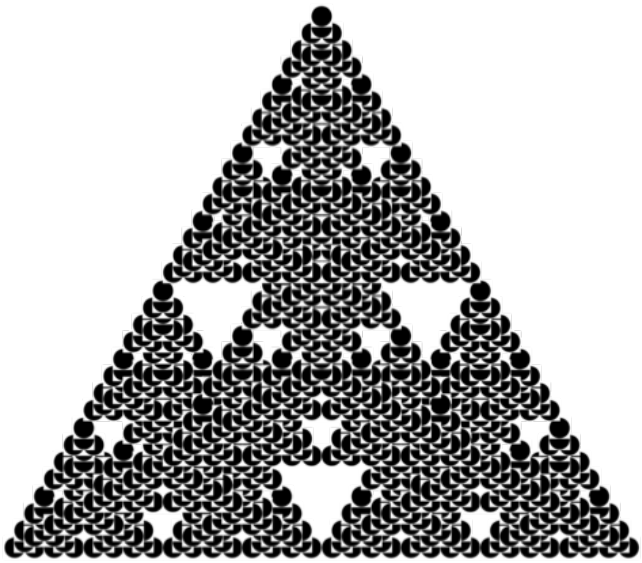
,



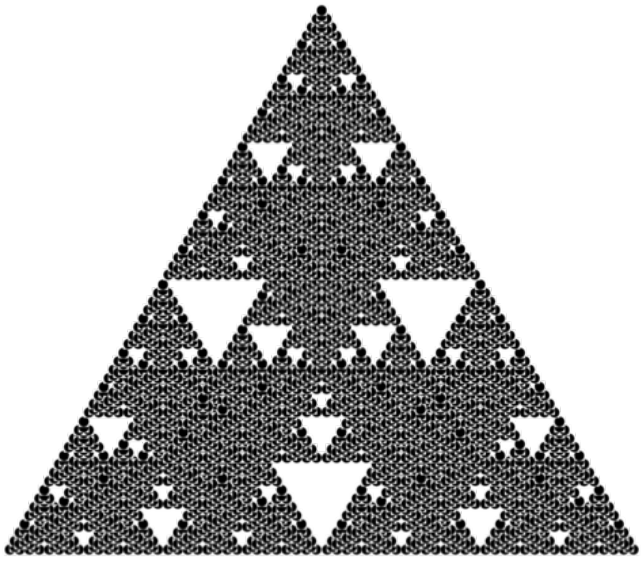
,



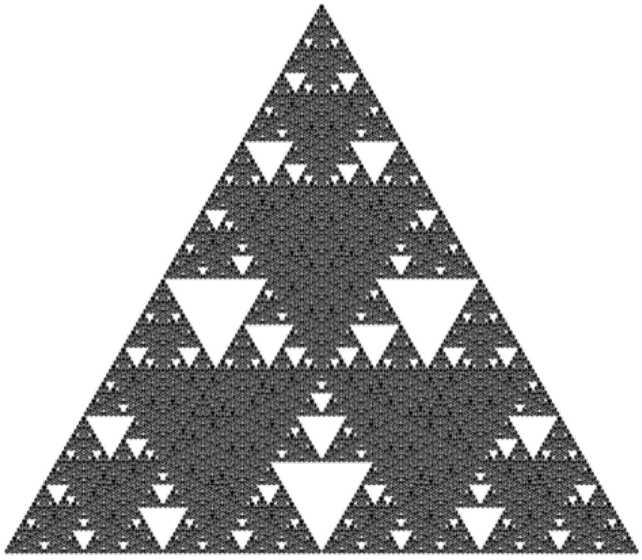
,



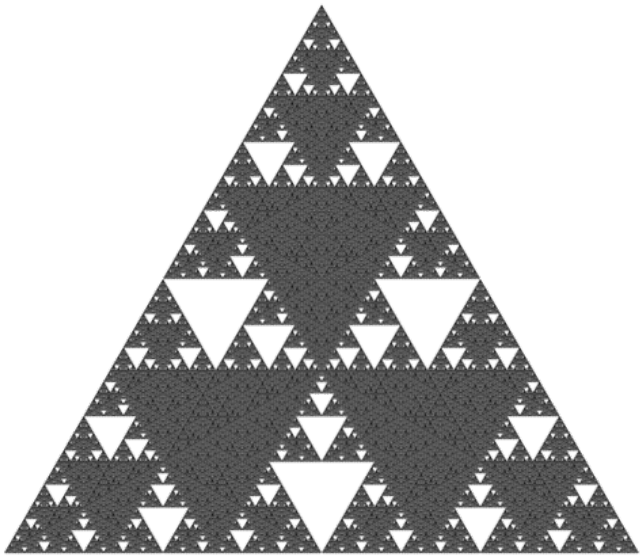
,



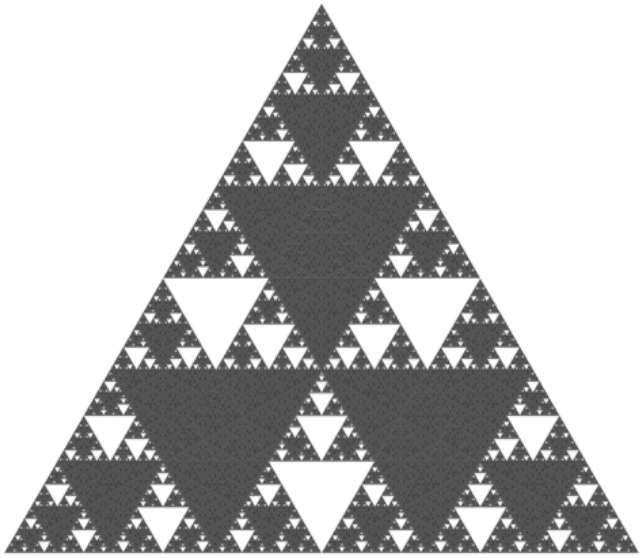
,



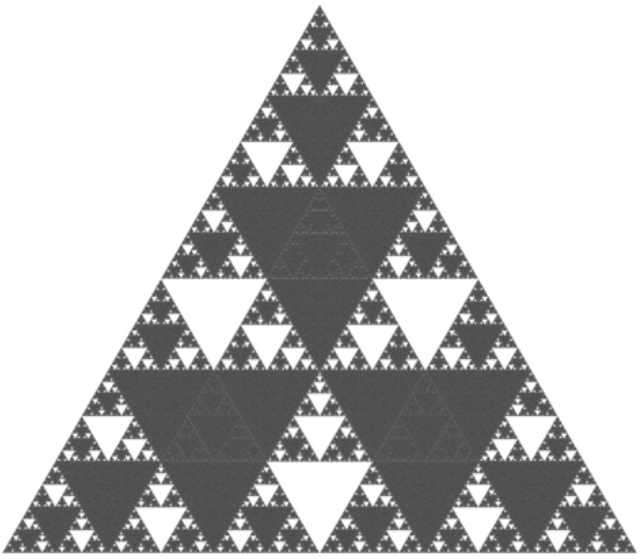
,



,



,

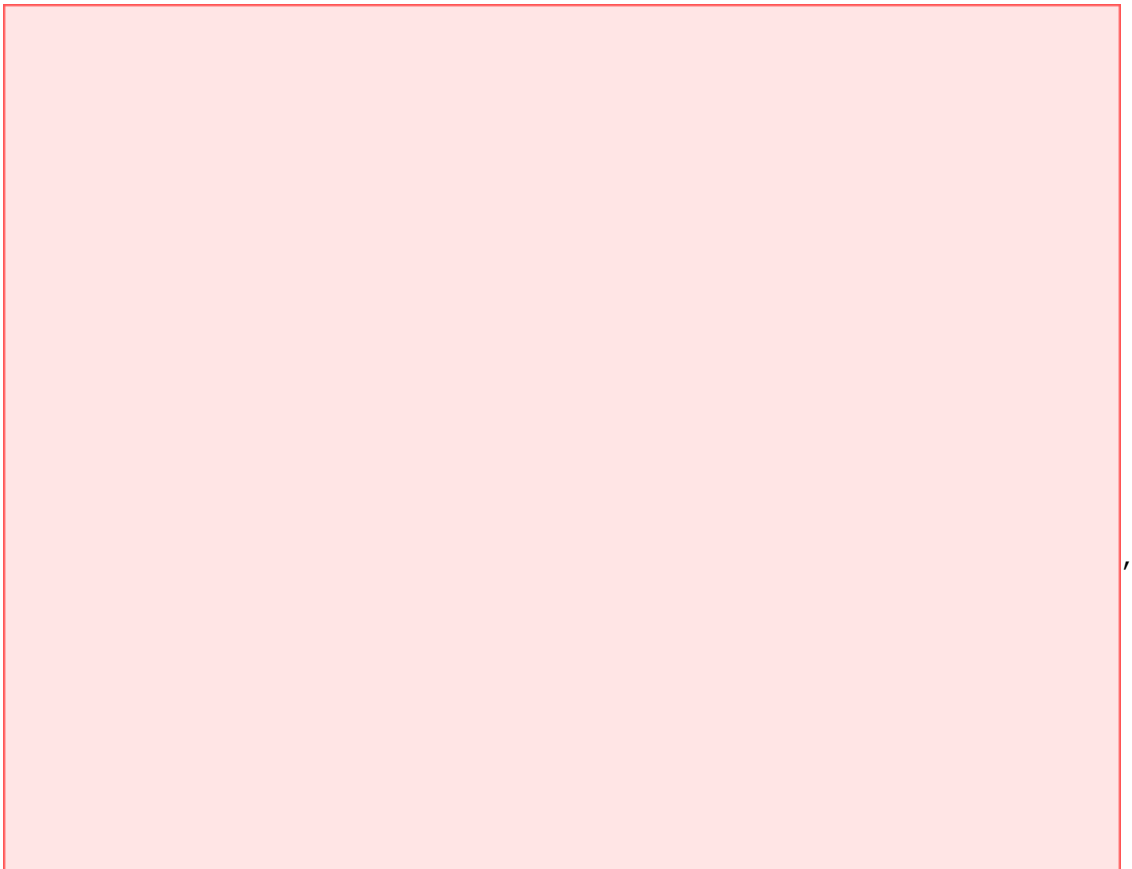


}

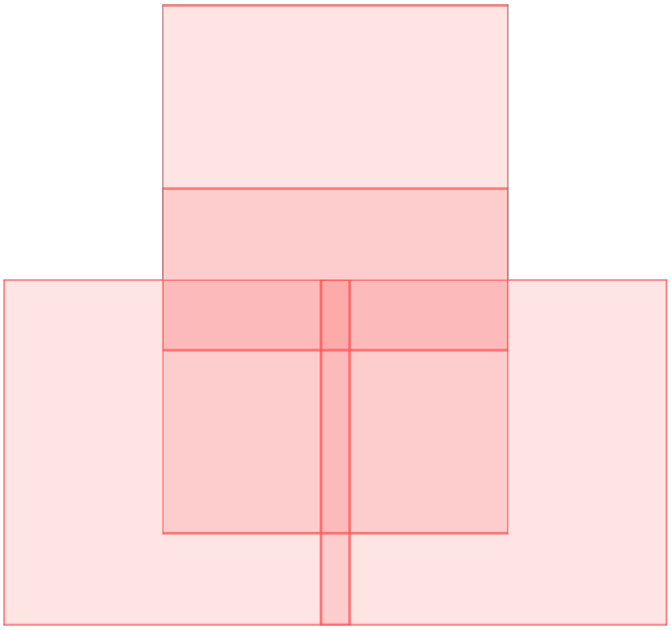
Mma's Pink Box of Graphic Humiliation (= PlotRage)

```
NestList[Rasterize[
   Style[Graphics[Table[{Translate[#[[1]], pts[[n]] * 330 / 256 * 256]}, {n, 1, 4}]],
    Antialiasing → False], RasterSize → 512, Background → None] &, Disk[], 10]
```

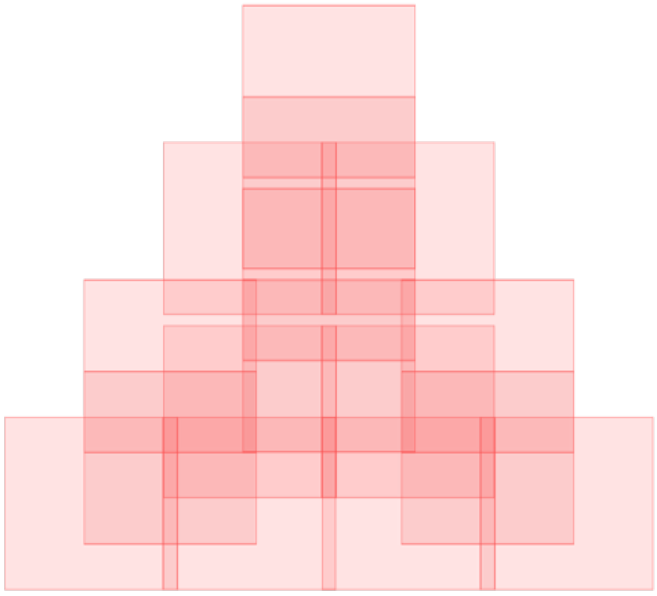
{Disk[{0, 0}],



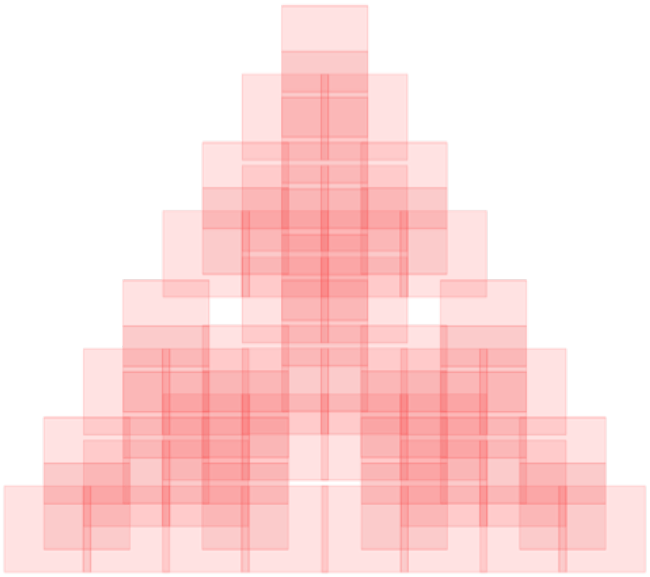
,



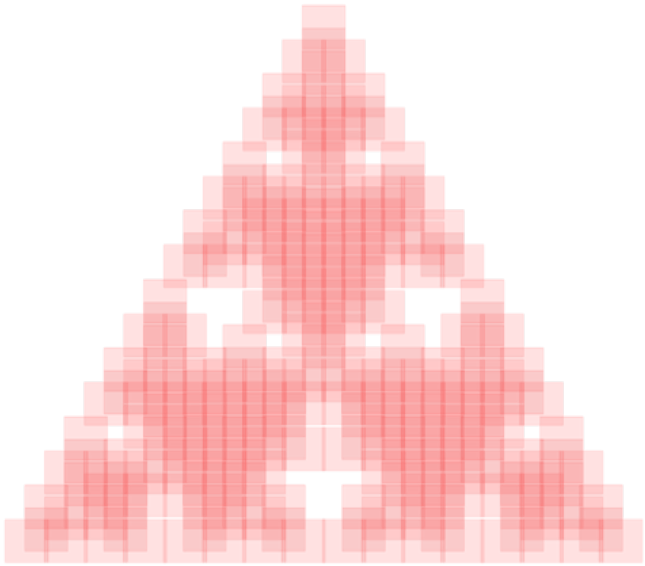
,



,

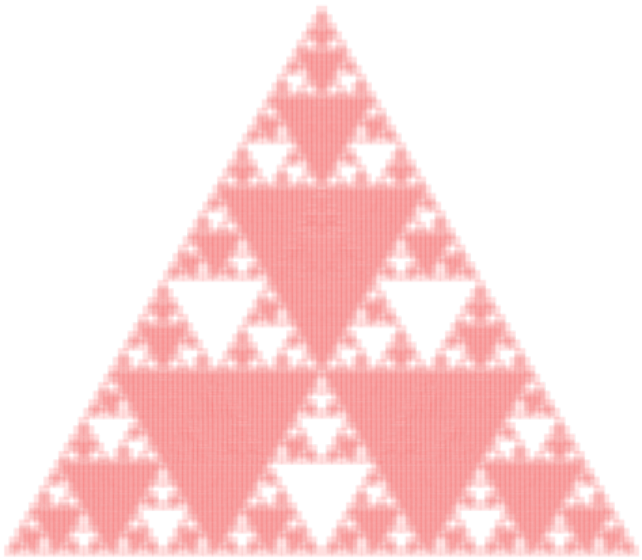


,

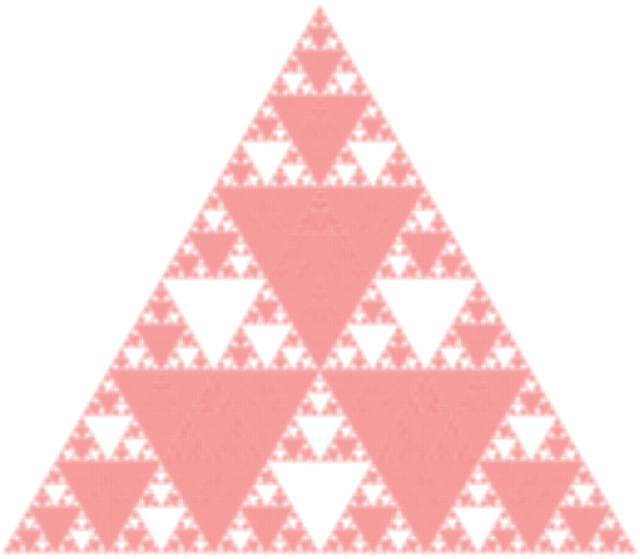


,

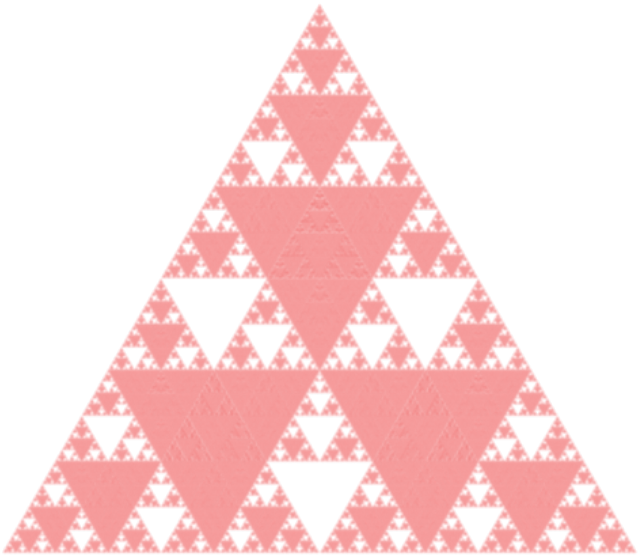
,



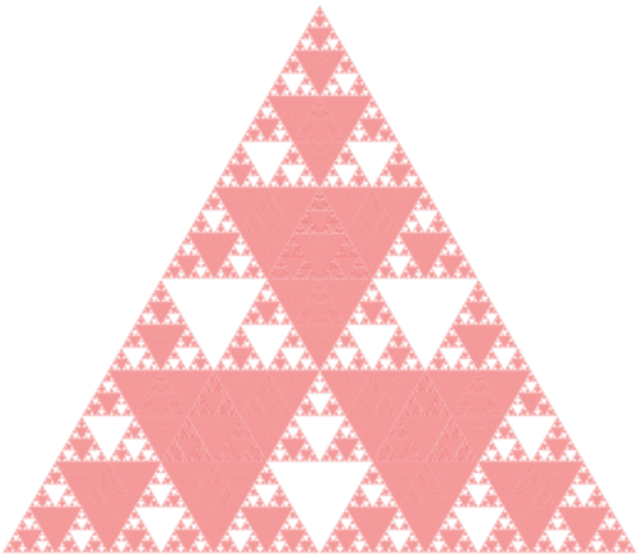
,



,



,



}

```
disco[0] = {}; disco[n_Integer] := Block[{di = disco[n - 1] / 3}, Join[di - 2 / 3, {I / 3}, di + 2 / 3]]
```

```
Graphics[{Line[{{-1, 0}, {1, 0}}], Opacity[.99, Yellow], Disk[{Re[#], 0}, Im[#]] & /@ disco[5]}]
```

```
%143 /. % → {1, 1, 1, 0} * 255
```