# Chebychyov = Lucas, Fibonacci

In[10]:=
```
2 * ChebyshevT[n, I / 2] / I^n == GoldenRatio^n + (-GoldenRatio)^-n == LucasL[n]
```

Out[10]=
$$2\,i^{-n}\,T_n\left(\frac{i}{2}\right) = (-\phi)^{-n} + \phi^n = L_n$$

In[11]:=
```
Simplify[Table[%, {n, -2, 5}]]
```

Out[11]=
$$\left\{\text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{GoldenRatio}^5 == 11 + \frac{1}{\text{GoldenRatio}^5}\right\}$$

In[12]:=
```
FullSimplify[%[[-1]]]
```

Out[12]=
```
True
```

This appears to extend to noninteger n, using $T_n$ := Cos(n ArcCos). As my tutor Julian noticed,

In[5]:=
```
Re[#[[1]]] == #[[2, 2]] &[%2 /. n → n + 1 / 2]
```

$$2\,\text{Re}\left(i^{-n-\frac{1}{2}}\,T_{n+\frac{1}{2}}\left(\frac{i}{2}\right)\right) = \phi^{n+\frac{1}{2}}$$

In[7]:=
```
FullSimplify[Table[%5, {n, -2, 5}]]
```

Out[7]=
```
{True, True, True, True, True, True, True, True}
```

We also have

In[8]:=
```
ChebyshevU[n, I / 2] / I^n == Fibonacci[n + 1]
```

Out[8]=
$$i^{-n}\,U_n\left(\frac{i}{2}\right) = F_{n+1}$$

In[9]:=
```
Simplify[Table[%, {n, -2, 5}]]
```

Out[9]=
```
{True, True, True, True, True, True, True, True}
```

which must be the world's shortest Fibonacci formula.
--Bill and Julian