Exact valuation of the Flowsnake (“Gosper curve”) spacefill

FlowS[t] gives the exact value for rational *t*. E.g.,

In[148]:=
```
FlowS /@ {0, 1 / 7, 1 / 3, 1}
```

Out[148]=
$$\left\{0, \frac{5}{14} - \frac{i\sqrt{3}}{14}, \frac{4}{7} + \frac{2i\sqrt{3}}{7}, 1\right\}$$

It’s a continuous function:

In[168]:=
```
FlowS /@ {7 / 22, 113 / 355}
```

Out[168]=
$$\left\{\frac{1693}{2762} + \frac{543\,i\sqrt{3}}{2762},\right.$$
$$2\,345\,845\,273\,398\,817\,840\,092\,950\,245\,183\,202\,891\,348\,356\,982\,612\,226\,750\,061\,526\,434\,654\,879 \backslash$$
$$728\,078\,331\,327\,658\,322\,166\,322\,544\,455\,096\,921\,202\,357 /$$
$$3\,824\,784\,042\,812\,146\,922\,648\,846\,082\,969\,066\,584\,366\,282\,400\,370\,622\,809\,380\,296\,235\,226\,226 \backslash$$
$$222\,138\,134\,929\,551\,565\,620\,475\,339\,497\,558\,916\,970\,979 +$$
$$(2\,172\,173\,105\,963\,516\,197\,779\,522\,804\,535\,911\,788\,205\,830\,019\,423\,847\,609\,465\,739\,564\,008\,821 \backslash$$
$$336\,204\,362\,398\,272\,785\,130\,819\,862\,477\,878\,379\,140\,827\,i) \Big/$$
$$\left.(3\,824\,784\,042\,812\,146\,922\,648\,846\,082\,969\,066\,584\,366\,282\,400\,370\,622\,809\,380\,296\,235\,226 \backslash$$
$$\left.226\,222\,138\,134\,929\,551\,565\,620\,475\,339\,497\,558\,916\,970\,979\sqrt{3}\right)\right\}$$

In[170]:=
```
N[%168]
```

Out[170]=
$$\{0.612962 + 0.340515\,i,\ 0.613328 + 0.327889\,i\}$$

We need this constant, whose magnitude is $1\Big/\sqrt{7}$:

In[32]:=
```
w = ComplexExpand[1 / (2 + (-1) ^ (1 / 3))]
```

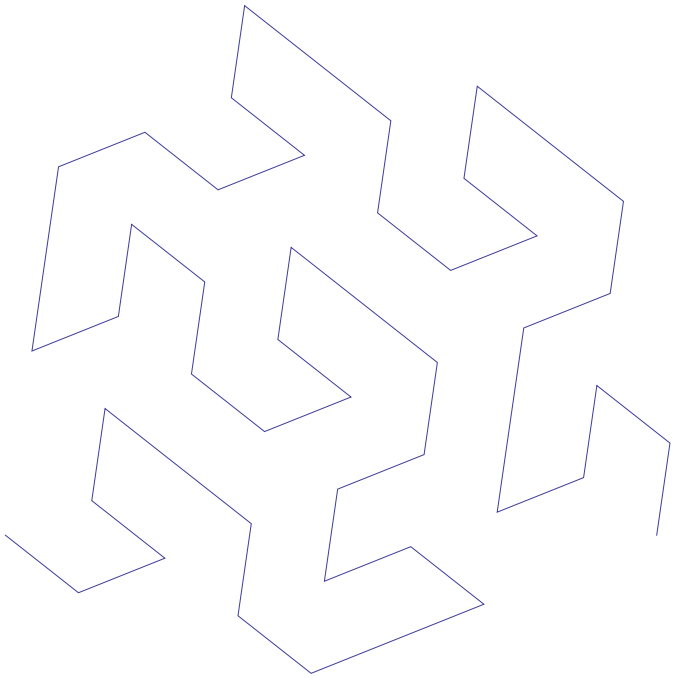
Out[32]=
$$\frac{5}{14} - \frac{i\sqrt{3}}{14}$$

As usual, FlowS redefines itself twice and has no obvious termination condition:

```
Clear[FlowS]; FlowS[t_, a_: 1, b_: 0] :=
 FlowS[t, x_: 0, y_: 0] = (FlowS[t, s1_: 1, s0_: 0] = (b - s0) / (s1 - a);
   Module[{u = t * 7, n}, u -= (n = Floor[u]);
    ComplexExpand[Switch[n,
      0, w * FlowS[u, a * w, b],
      1, w * (1 + (-1)^(1 / 3) +
         (-1)^(4/3) * FlowS[1 - u, a * (-1)^(4 / 3) * w, b + a * w * (1 + (-1)^(1 / 3))]),
      2, w * ((-1)^(1 / 3) + FlowS[1 - u, a * w, b + a * w * (-1)^(1 / 3)]),
      3, (*Typo fixed*)w * ((-1)^(1 / 3) +
         (-1)^(2 / 3) * FlowS[u, a * (-1)^(2 / 3) * w, b + a * w * (-1)^(1 / 3)]),
      4, w * (Sqrt[-3] + FlowS[u, a * w, b + a * w * Sqrt[-3]]),
      5, w * (2 * (-1)^(1 / 3) + FlowS[u, a * w, b + a * w * 2 * (-1)^(1 / 3)]),
      6, w * (2 + (-1)^(1 / 3) + ((-1)^(1 / 3) - 1) *
          FlowS[1 - u, a * w * ((-1)^(1 / 3) - 1), b + a * w (2 + (-1)^(1 / 3))]),
      7,
      1]]])
```

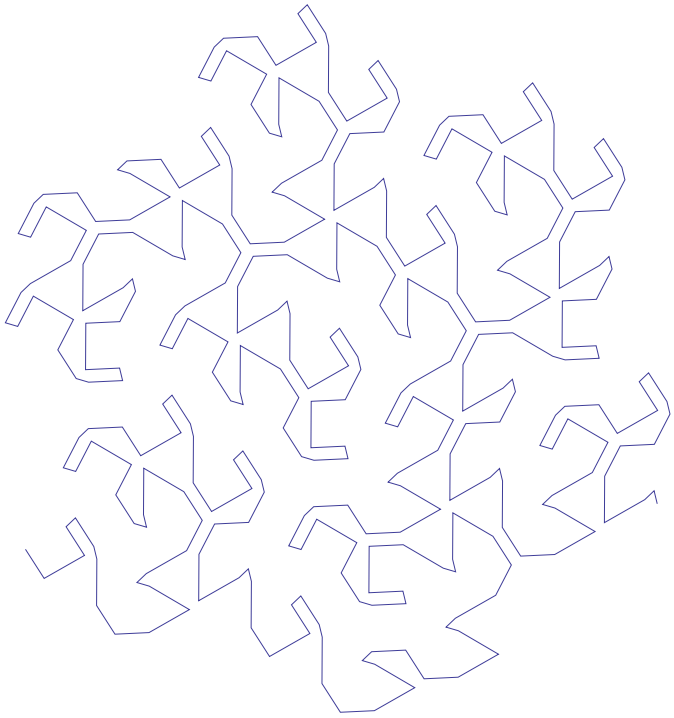
In[147]:=
```
ListLinePlot[{Re[#], Im[#]} & /@ (FlowS /@ Range[0, 1, 1 / 49]),
 Axes → False, AspectRatio → Automatic]
```

Out[147]=



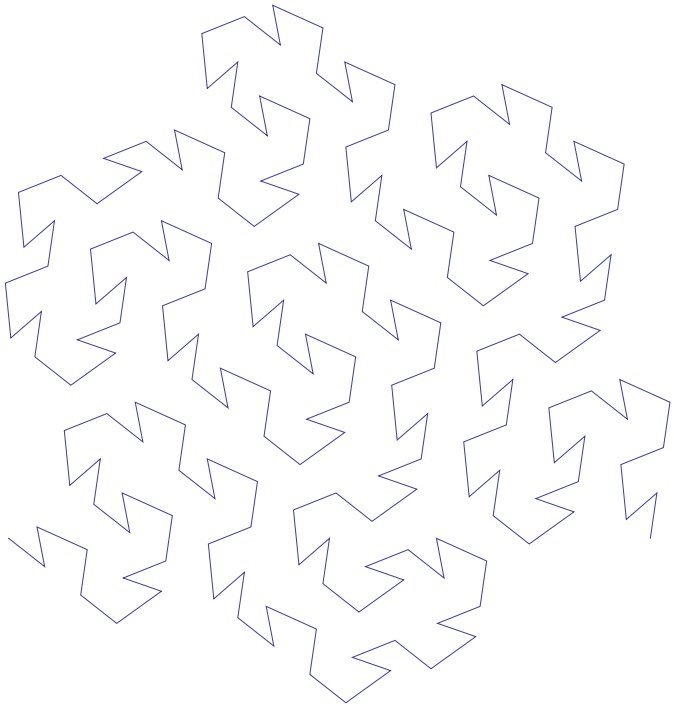
In[145]:=
```
ListLinePlot[{Re[#], Im[#]} & /@ (FlowS /@ Range[1 / 49 / 49, 1, 1 / 49 / 7]),
 Axes → False, AspectRatio → Automatic]
```

Out[145]=



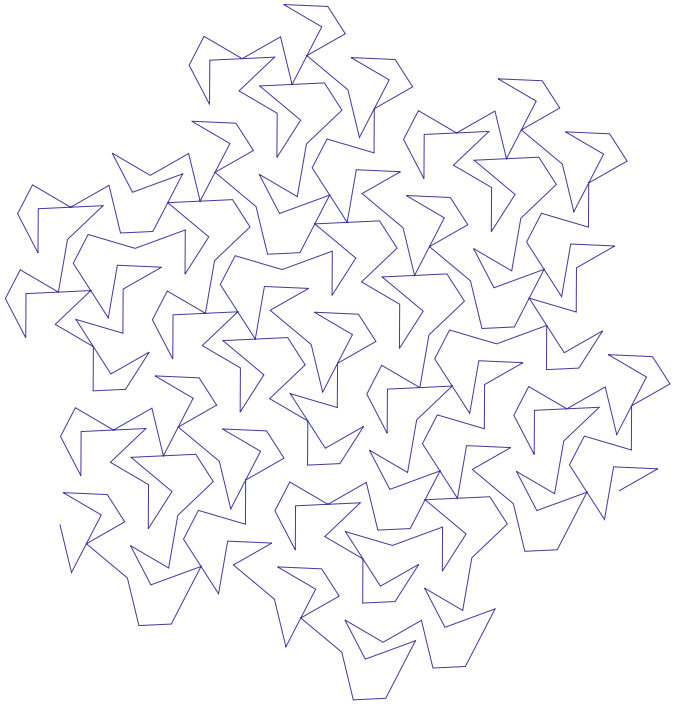
In[158]:=
```
ListLinePlot[{Re[#], Im[#]} & /@ (FlowS /@ Range[0, 1, 1 / 49 / 4]),
 Axes → False, AspectRatio → Automatic]
```

Out[158]=



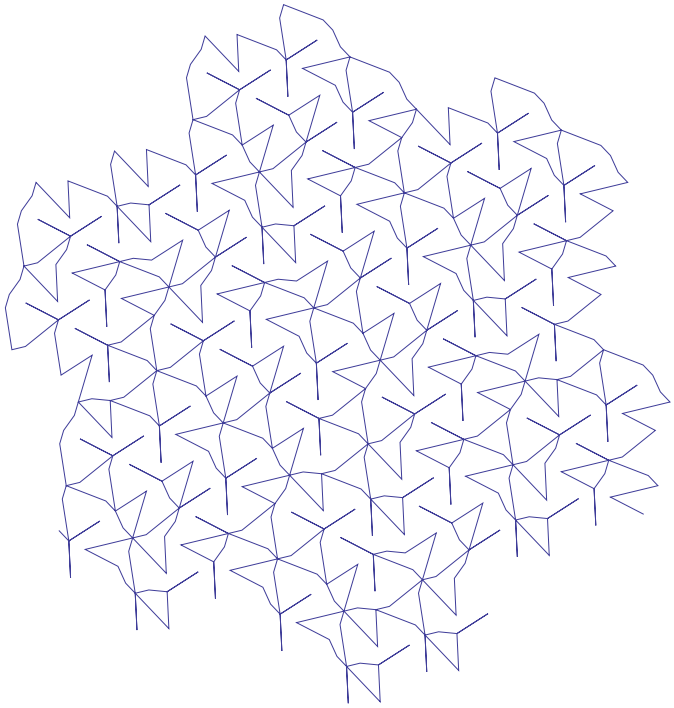
In[137]:=
```
ListLinePlot[{Re[#], Im[#]} & /@ (FlowS /@ Range[1 / 49 / 3 / 7, 1, 1 / 49 / 7]),
 Axes → False, AspectRatio → Automatic]
```

Out[137]=



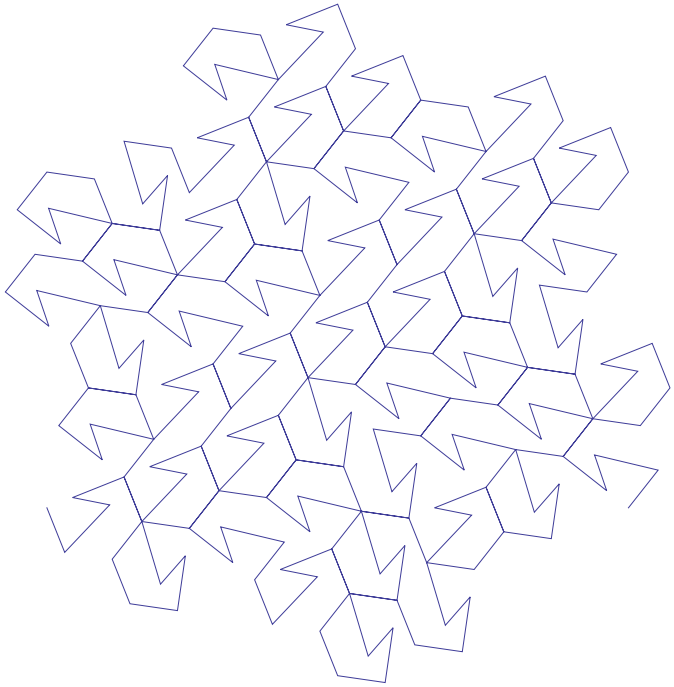
In[139]:=
```
ListLinePlot[{Re[#], Im[#]} & /@ (FlowS /@ Range[1 / 49 / 7 / 3, 1, 1 / 49 / 7 / 2]),
 Axes → False, AspectRatio → Automatic]
```

Out[139]=



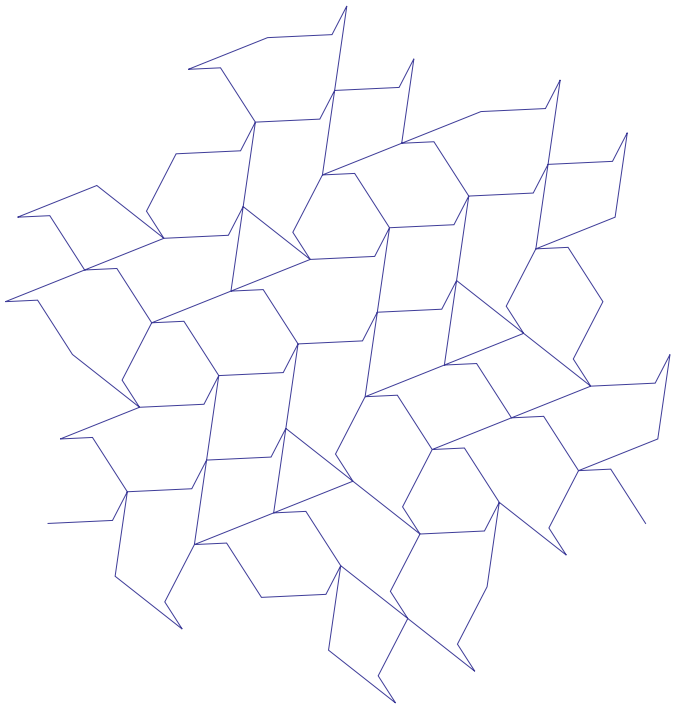
In[160]:=
```
ListLinePlot[{Re[#], Im[#]} & /@ (FlowS /@ Range[0, 1, 1 / 49 / 6]),
 Axes → False, AspectRatio → Automatic]
```

Out[160]=



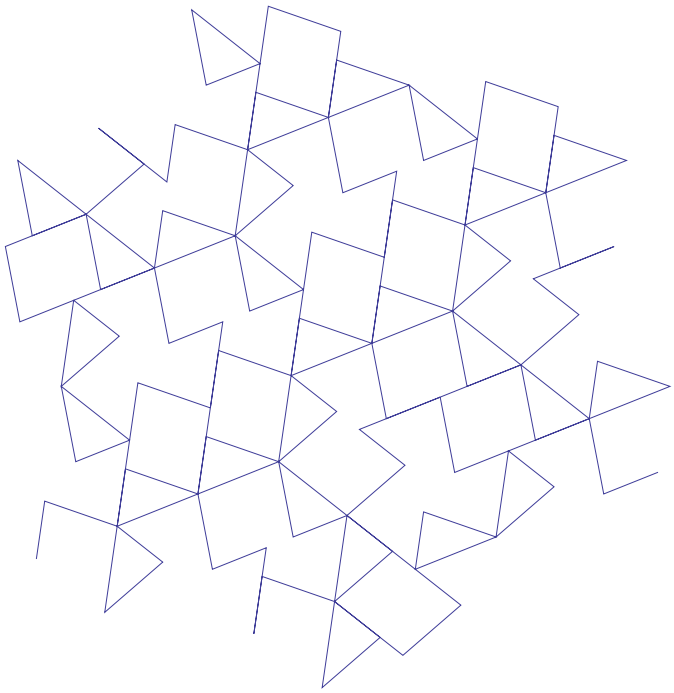
In[144]:=
```
ListLinePlot[{Re[#], Im[#]} & /@ (FlowS /@ Range[0, 1, 1 / 49 / 3]),
 Axes → False, AspectRatio → Automatic]
```

Out[144]=



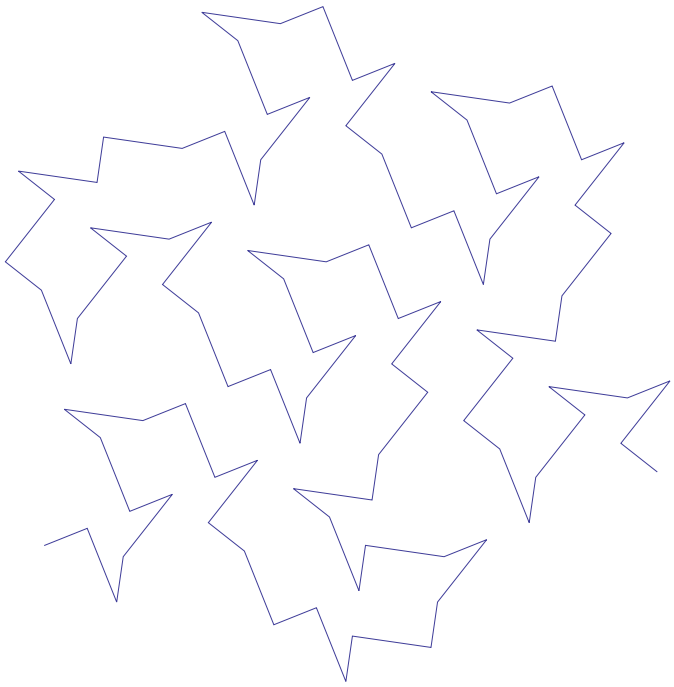
In[157]:=
```
ListLinePlot[{Re[#], Im[#]} & /@ (FlowS /@ Range[1 / 49 / 6, 1, 1 / 49 / 3]),
 Axes → False, AspectRatio → Automatic]
```

Out[157]=



In[167]:=
```
ListLinePlot[{Re[#], Im[#]} & /@ (FlowS /@ Range[1 / 49 / 4, 1, 1 / 49 / 2]),
 Axes → False, AspectRatio → Automatic]
```

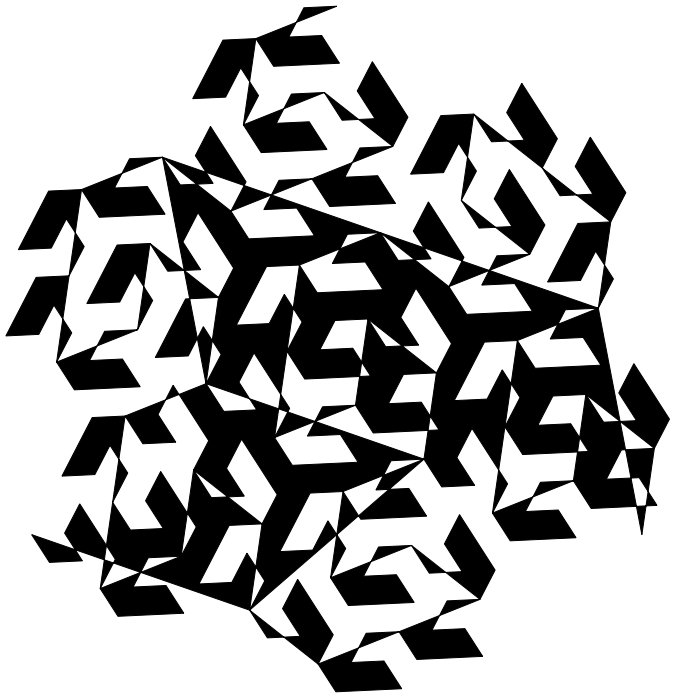
Out[167]=



In[190]:=
```
trifl[a_, b_, c_] :=
 Graphics[Polygon[{Re[#], Im[#]} & /@ Join[FlowS /@ a, FlowS /@ b * (-1)^(2 / 3) + 1,
     FlowS /@ c * (-1)^(4 / 3) + (-1)^(1 / 3)]]]
```

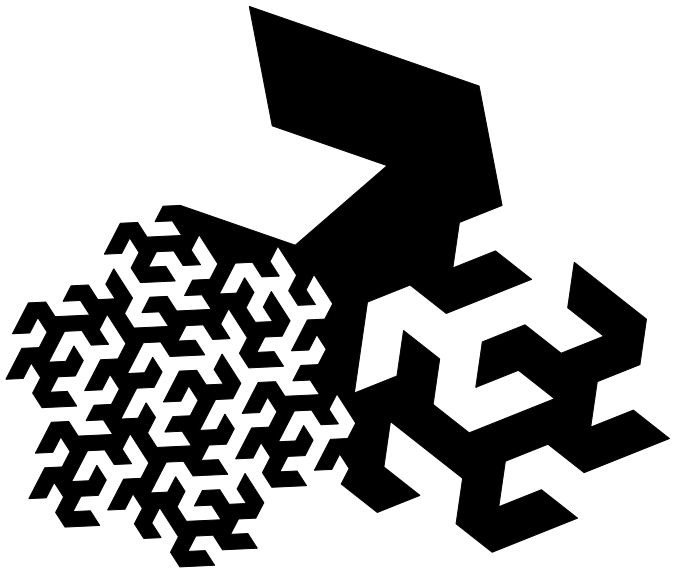
In[191]:=
```
trifl[Range[0, 1, 1 / 7], Range[0, 1, 1 / 49], Range[0, 1, 1 / 343]]
```

Out[191]=



In[194]:=
```
trifl[Range[0, 1, 1 / 7], Range[0, 1, 1 / 49], Range[0, 1, 1 / 343]]
```

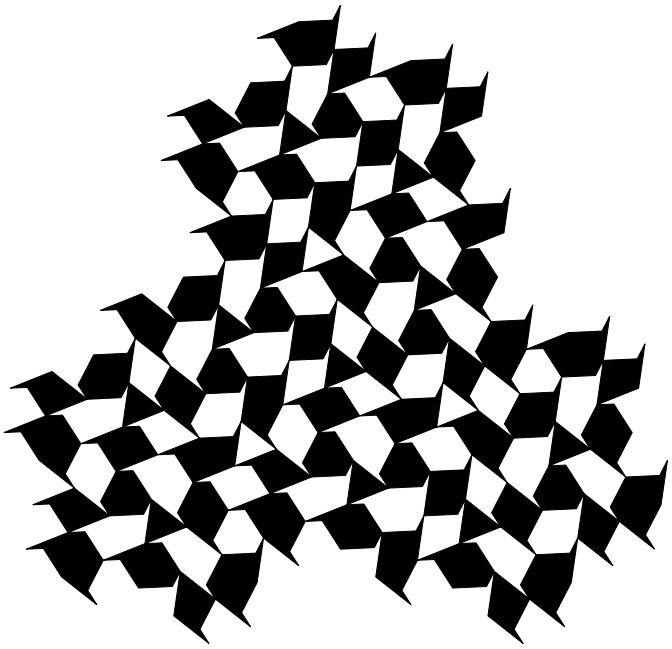
Out[194]=



In[193]:=
```
trifl[a_, b_, c_] :=
 Graphics[Polygon[{Re[#], Im[#]} & /@ Join[FlowS /@ a, FlowS /@ b * (-1)^(4 / 3) + 1,
     FlowS /@ c * (-1)^(2 / 3) + (-1)^(5 / 3)]]]
```

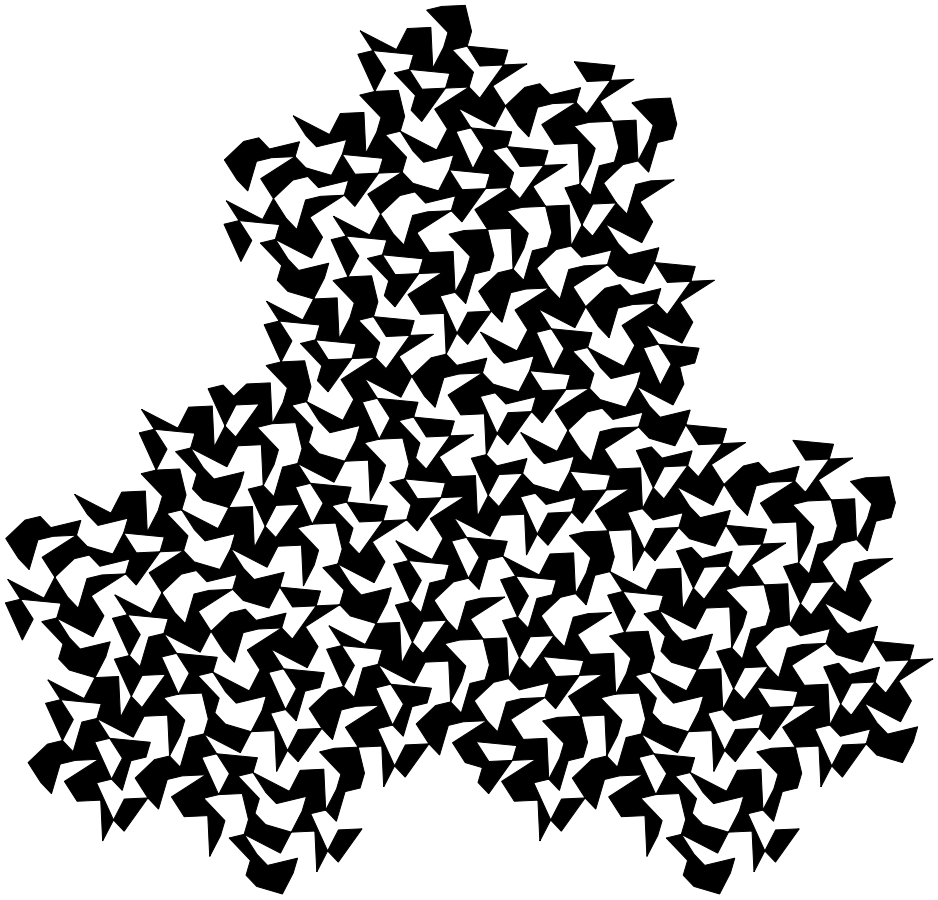
In[195]:=
```
trifl[#, #, #] &@@Range[0, 1, 1 / 49 / 3]
```

Out[195]=



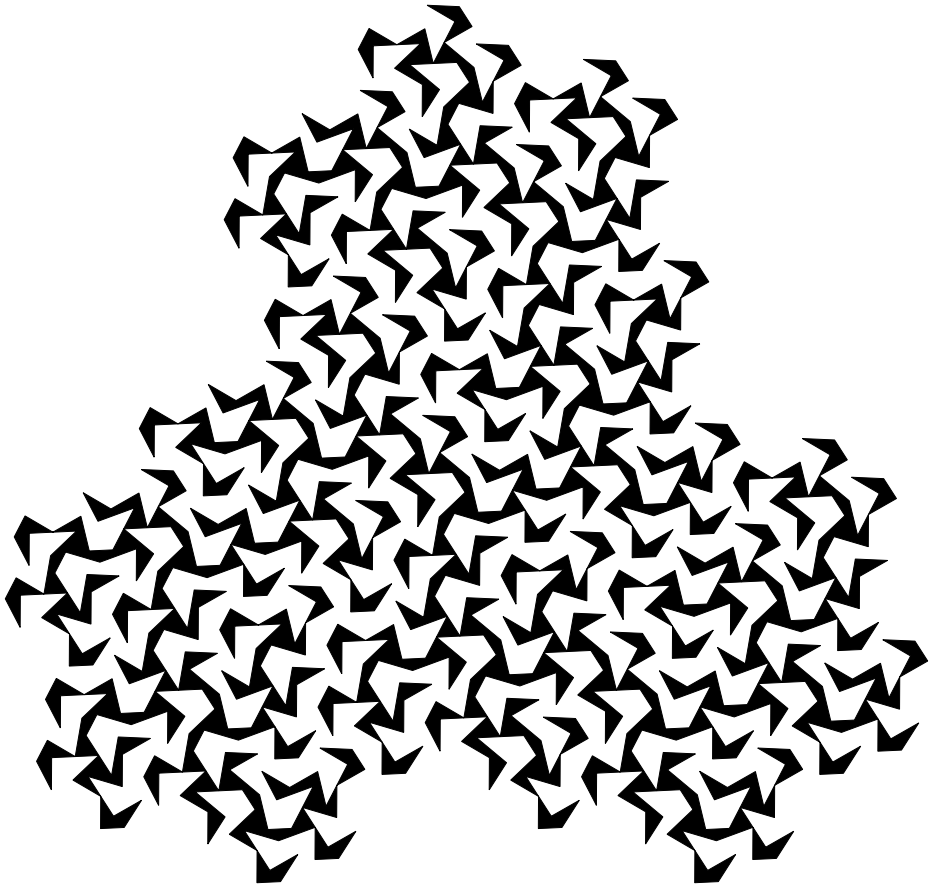
In[196]:=
```
trifl[#, #, #] &@@Range[0, 1, 2 / 49 / 3 / 7]
```

Out[196]=



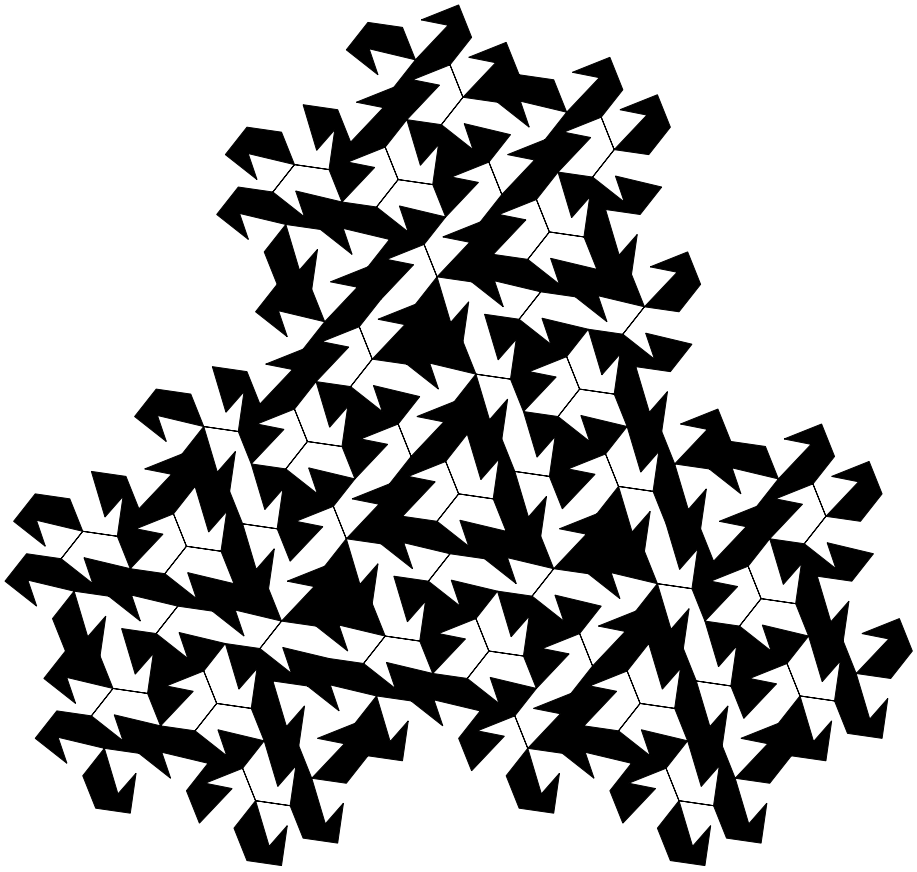
In[197]:=
```
trifl[#, #, #] &@@Range[1 / 49 / 3 / 7, 1, 1 / 49 / 7]
```

Out[197]=



In[198]:=
```
trifl[#, #, #] &@@Range[0, 1, 1 / 49 / 6]
```

Out[198]=



In[199]:=
```
trifl[#, #, #] &@@Range[1 / 49 / 6, 1, 1 / 49 / 3]
```

Out[199]=