Solid Geometry Formulas

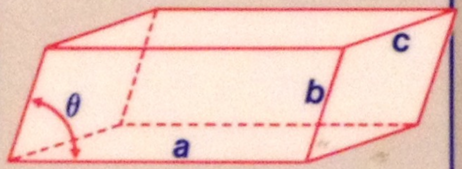
Paperstech

Full Size Edition
Reference Guide

PLANE SOLIDS

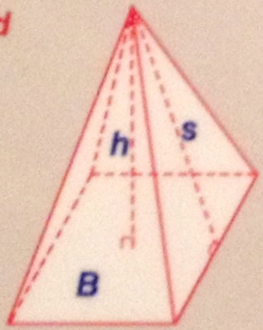
Parallelepiped

	θ	Area	Volume
General	0–180°	$2(ac + bc + ab\sin\theta)$	$abc\sin\theta$
Rectangle	90°	$2(ab + bc + ac)$	abc
Cube ($a = b = c$)	90°	$6a^2$	a^3



Right Regular Pyramid

B = base area
P = base perimeter
s = slant height
$\text{Area}_L = \frac{1}{2}Ps$
$\text{Area}_T = \frac{1}{2}Ps + B$
$\text{Volume} = \frac{1}{3}Bh$
Altitude h intersects center of base



Prismatoid

Definition: A polyhedron consisting of either two parallel polygon bases or a polygon base and vertex. The lateral faces consist of either trapezoids or triangles respectively.

B_1 = lower base area
B_2 = upper base area
M = midsection area
h = altitude
$\text{Volume} = \frac{1}{6}h(B_1 + B_2 + 4M)$

Regular Polyhedra

Definition: A regular polyhedron has equal dihedral angles and faces which are congruent regular polygons.

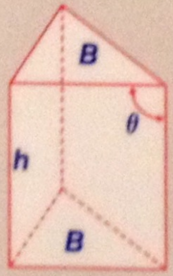
f = number of faces
e = number of edges
v = number of vertices
r = radius of inscribed sphere
A = face area
θ = dihedral angle
a = edge length

Euler-Descartes formula: $v - e + f = 2$ $\text{Area}_T = fA$ $\text{Volume} = \frac{1}{3}rfA$

Name	Face	v	e	f	θ	Area_T	Volume
Tetrahedron	4 equilateral triangles	4	6	4	70° 32′	$1.7321\,a^2$	$0.1178\,a^3$
Hexahedron	6 squares	8	12	6	90°	$6.0000\,a^2$	$1.0000\,a^3$
Octahedron	8 equilateral triangles	6	12	8	109° 28′	$3.4641\,a^2$	$0.4714\,a^3$
Dodecahedron	12 regular pentagons	20	30	12	116° 34′	$20.6457\,a^2$	$7.6631\,a^3$
Icosahedron	20 equilateral triangles	12	30	20	138° 11′	$8.6602\,a^2$	$2.1817\,a^3$

Right Regular Prism

B = base area
P = base perimeter
$\text{Area}_L = Ph$
$\text{Area}_T = Ph + 2B$
$\text{Volume} = Bh$

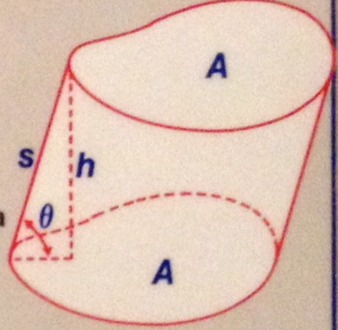


For all sections:
Area_L = lateral surface area
Area_T (or Area) = total surface area

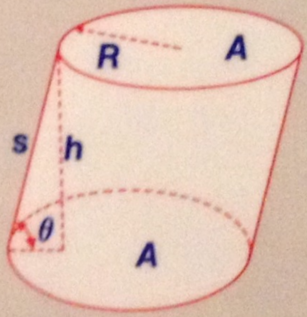
CYLINDERS

General Cylinder

$\text{Area}_L = Ps$
$= Ph/\sin\theta$
$\text{Volume} = Ah$
$= As\sin\theta$
A represents the area of any closed curve.



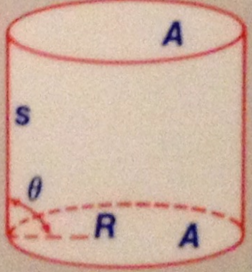
Circular Cylinder



$A = \pi R^2$
$P = 2\pi R$
$\text{Area}_L = 2\pi Rs$
$= 2\pi Rh/\sin\theta$
$\text{Volume} = \pi R^2 h$
$= \pi R^2 s\sin\theta$

Right Circular Cylinder

$\theta = 90°$ $A = \pi R^2$
$h = s$ $P = 2\pi R$
$\text{Area}_L = 2\pi Rh$
$\text{Volume} = \pi R^2 h$



For all sections:
A = base area P = base perimeter

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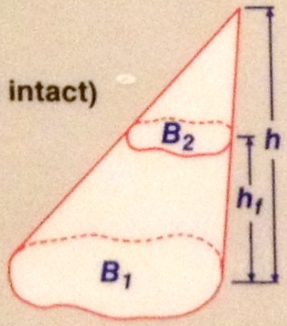
CONES

General Cone

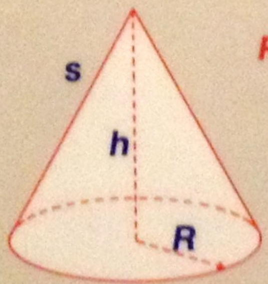


B_1 = lower base area
B_2 = 0 (i.e. tip of cone intact)
h = altitude
Volume = $\frac{1}{3}B_1h$

Frustrum of General Cone

B_1 = lower base area
B_2 = upper base area
h_f = distance between parallel bases B_1 and B_2

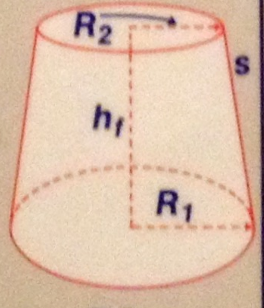
Volume = $\frac{1}{3}h_f(B_1 + B_2 + \sqrt{B_1B_2})$



Right Circular Cone

$s = \sqrt{R^2 + h^2}$ (slant height)
$\text{Area}_L = \pi Rs$
$\text{Area}_T = \pi R(R + s)$
Volume = $\frac{1}{3}\pi R^2h$

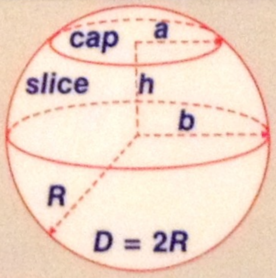
Frustrum of Right Circular Cone



$s = \sqrt{(R_1 - R_2)^2 + h_f^2}$ (slant height)
h_f = distance between upper and lower bases
$\text{Area}_L = \pi(R_1 + R_2)s$
$\text{Area}_T = \pi[R_1^2 + R_2^2 + (R_1 + R_2)s]$

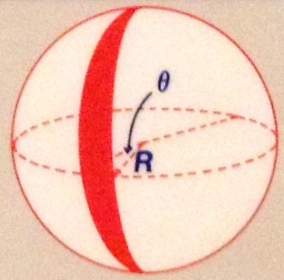
Volume = $\frac{1}{3}\pi h_f(R_1^2 + R_1R_2 + R_2^2)$

SPHERES AND SPHERIODS



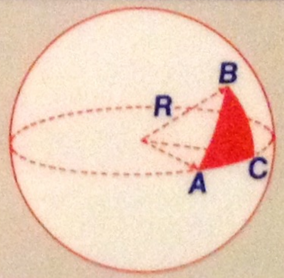
Sphere

Area = $4\pi R^2 = \pi D^2$
$= 12.5664R^2$
Volume = $\frac{4}{3}\pi R^3 = \frac{1}{6}\pi D^3$
$= 4.1888R^3$



Lune

θ in radians
Area (shaded) = $2R^2\theta$
Volume = $\frac{2}{3}\theta R^3$



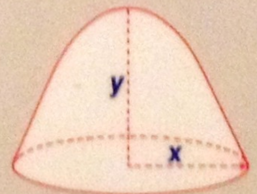
Spherical Triangle

A, B, and C in radians
Area = $(A + B + C - \pi)R^2$

Spherical Cap

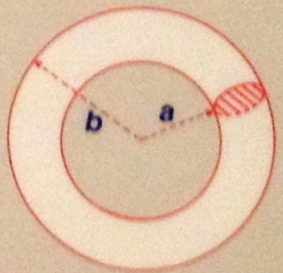
Formed by making an arbitrary slice through a sphere

a = radius of cap base
Area (cap) = $2\pi Rh$
Volume = $\frac{1}{3}\pi h^2(3R - h)$
$= \frac{1}{6}\pi h(3a^2 + h^2)$



Paraboloid

Volume = $\frac{1}{2}\pi a^2b$



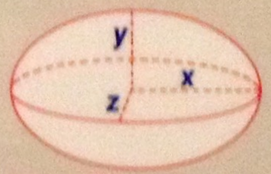
Circular Torus

R = radius of cross section
Area = $\pi^2(b^2 - a^2)$
$= 4\pi^2bR$
Volume = $\frac{1}{4}\pi^2(a + b)(b - a)^2$
$= 2\pi^2bR^2$

Spherical Slice

Formed by making two parallel slices through a sphere

a = radius of upper base
b = radius of lower base
Area (surface) = $2\pi Rh$
Volume = $\frac{1}{6}\pi h(3a^2 + 3b^2 + h^2)$



Ellipsoid

Volume = $\frac{4}{3}\pi abc$

In the following: a = major semiaxis b = minor semiaxis ϵ = eccentricity = $\sqrt{\frac{a^2 - b^2}{a}}$

Prolate Spheroid (Oblong-shaped)

Formed by rotating ellipse about major axis

Area = $2\pi a^2 + \frac{\pi b^2}{\epsilon}\ln\left(\frac{1+\epsilon}{1-\epsilon}\right)$

Volume = $\frac{4}{3}\pi a^2b$

Oblate Spheroid (Disc-shaped)

Formed by rotating ellipse about minor axis

Area = $2\pi b^2 + \frac{2\pi ab}{\epsilon}\sin^{-1}\epsilon$

Volume = $\frac{4}{3}\pi ab^2$

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